A Multi-attribute Auction Model by Dominance-based Rough Sets Approach

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Abstract. As an alternative to the price-based traditional auction model, the multi-attribute auction model is an integrated model requiring the simultaneous trade of different types of attributes as the sellers and buyers deal. As a result, the design and modeling of the auction mechanism have become very difficult. This paper proposes a multi-attribute auction model using the dominance-based rough sets approach (DRSA). The multi-attribute decision method by DRSA can directly mine out the preference relations between the attributes of alternatives so that relevant auction mechanisms can be designed. This model uses a natural reasoning procedure similar to that of decision makers. Finally, a numerical example demonstrates the simplicity, efficiency, and feasibility of the proposed auction model.

Keywords: ulti-attribute decision, rough sets, auction, DRSA.

1. Introduction

An auction is an ancient pricing mechanism that began with Chaldeans(fl. c.340-c.270 B.C.) auctioning their wives, ancient Egyptians auctioning their mining rights, and ancient Romans auctioning their slaves, booties, and debtor's belongings. Research on auction theories began very late, and quantitative studies on auction problems did not commence until the 1960s. Further, although the quantitative study of auction problems is a very complex

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process, many interesting conclusions have been reached. For instance, researchers found that an auction is a perfect price discovering mechanism when both the supplier and the buyer are having difficulties in naming a price. The American FCC(Federal Communications Commission) spectrum auction, known as the greatest auction in history, has become the foundation of spectrum auctions in many regions. It has resolved the disjointedness of the auction theory and auction practice, while proving the practicability of the game theory. Furthermore, the auction theory can be used to explain many complex non-economic phenomena (Klemperer 2006).

A traditional auction is the public sale of a product based on price bidding, wherein the highest bidder acquires the product. The seller wants to sell at a greater range of prices, while the buyer wants to buy at a smaller range of prices. Traditional auction problems have been analyzed by the game theory approach for a long time. New auction models have emerged because of developments in the society, especially the advent of information technology. For example, the procurement competition model, which is often used by the US Department of Defense, is unlike most traditional auction models and involves many performance/quality dimension data aside from price, such as promised technical characteristics, delivery date, and managerial performance (Che 1993). The evaluation of bids in a multi-attribute auction involves the application of an elaborate scoring system designed by the buyer: each individual component of a bid is evaluated and assigned a score, the scores are summed to yield a total score, and the firm achieving the highest total score wins the contract.

The use of the Internet as a platform has allowed the inclusion of many novel and useful features in auctions. One of these features is the incorporation of multiple attributes, which is based on the realization that there is much more to value than price (Teich et al. 2006). By the B2B mode, multi-attribute online auction is a new type of auction on the Internet that has risen with the development of e-commerce in recent years. Through the dominance of information in the Internet, online auctions can help both the buyer and the seller in expanding their range for finding individuals to cooperate with and in ensuring that an approved price for both sides is settled on. Recent trends have required that a product cannot be auctioned simply by its price. Instead, the multi-attribute auction, which is based on the multiple attributes of a product, such as its price, quality, time of delivery, and management performance of the supplier, has become de rigeur. At present, multi-attribute online auctions have been applied in public biddings of government projects and in the procurement of enterprises. Thus, multi-attribute trading mechanisms transcend traditional, price-only mechanisms by facilitating a negotiation over a set of predefined attributes representing various non-price aspects of the product (Engel and Wellman 2007).

In this paper, a novel multi-attribute online auction model is developed based on the dominance-based rough set approach (DRSA). Similar to the natural reasoning learning method, the new method can reuse the agent's preferences to greatly increase bidding efficiency.

The rest of this paper is organized as follows. Section 2 is the literature review. Section 3 will advance the multi-attribute decision model by dominance-based rough sets approach. Section 4 will develop a new-type multi-attribute auction model by dominance-based rough sets approach, Section 5 is an example, and the last section provides concluding remarks.

2.   Related Literature

Che (1993), the first to study the multi-attribute auction problem, designed a score rule for integrating two attributes and the auction mechanism, and proved the two-dimensional revenue equivalence theorem. Branco (1997) extended Che's study by taking into account auction problems correlated with the firm's cost, and concluded that the optimal effect cannot be directly obtained through the auction process but by a two-stage mechanism. Bichler (2000) carried out experimental analyses of the similar utility function and found that multi-attribute auctions yield utility scores that are significantly higher than those of single-attribute auctions. Based on two attributes and existing auction algorithms, Teich et al. (2000a, 2006b) described a Java-based Web auction system. They designed a "suggested price" mechanism and introduced its algorithms and theoretical foundations. Mishra and Veeramani (2002) identified multi-attribute auction problems in the outsourcing industry. They developed a simple and feasible auction mechanism proven to lead to competitive equilibrium while estimating the private cost functions of suppliers and the private valuation functions of the parent-firms based on the attribute values to be known. Karimi et al. (2007) stated that multi-attribute procurement problems have different characteristics under the supply chain framework. The exact value of the production cost is kept private and is difficult to be known by producers and customers. They designed an auction mechanism that integrated two attributes (time and price) into the scoring rule. Engel and Wellman (2007) acknowledged that trader's preferences cannot be ignored and that the full additivity of attributes cannot be assumed. Therefore, they introduced an iterative auction mechanism that maintained the prices in local clusters related to attributes rather than the full space of joint configurations. Jin et al (2006a, 2006b) introduced the MAV(Multi-Attribute auction model pioneered by Vickrey) and MAE(Multi-Attribute auction model pioneered by Esther David's) models for multi-attribute auctions and compared them with existing models to demonstrate the advantages of the new models. However, they did not constrain the number of attributes, and they assumed that both the cost functions of the seller and the valuation functions of the buyer are known. Another study (Wang et al. 2006) used the incentive mechanism to design a franchise bidding mechanism for a regional district distribution service to maximize the expected social welfare. However, the format of the optimal bidding mechanism is so complicated that a two-stage practical application is needed to implement it, namely, to bid first and then to negotiate about the quality.

Most extant studies often integrate multidimensional attributes to two-dimensional attributes, including price and quality, by first predigesting multiple attributes (except for price) into a quality attribute. Then the characteristics of the commodity are mapped from multidimensional to one-dimensional real number space by the valuation function, score rule, cost function, or social welfare function. Finally, the multi-attribute auction format is studied using the game theory and object optimization. However, the auction models developed from two-dimensional attributes are already complex and difficult to implement in practical applications. In management applications, the cost and social welfare functions are private for the seller and the buyer, so that the mapping from multidimensional space to single dimension space is extremely focal and difficult. Most studies examine the multi-attribute auction from the view of mechanism design, generally predigesting this kind of problem with the assumption that the valuation function is known and failing to consider it as the focus of the study.

The design of scoring rules for multiple attributes and the identification of the influence of different scoring rules on auction performance are among the most important study directions of multi-attribute auction problems. Based on the multi-attribute decision theory, multi-attribute auction studies should not constrain the number and types of attributes (whether qualitative or quantitative) of the commodity. A study (Wu, 2007) proposed the use of the multi-attribute decision making of the weighted aggregative valuation method to choose the winning bidder and introduced full bidding rules. During multiple rounds of bidding, suppliers and buyers are permitted to continually adjust the weights of attributes and focus on the importance of the assignment of weights. Xie and Li (2005) evaluated bidding alternatives in multi-attribute online auction decisions with the fuzzy aggregative valuation method. They also made attribute reductions and set the weights of attributes using the rough set theory. Other than these two methods, the rest of studies proposed that multiple attributes should be aggregated first before evaluating bidding based on aggregative valuation.

Other studies also paid more attention to agent preferences. In multiple rounds of bidding and negotiation under limited time, buyers or sellers may follow their own preferences. If the preferences of the selected winner in the previous round can be extracted, they can be used to predict the optimal bidding in the next round, thus increasing substantially the efficiency of the multi-attribute auction. At present, the rough set method is preferred for extracting decision preferences. Furthermore, each attribute in a multi-attribute auction has ordinal properties in addition to classifiable characteristics. Hence, based on the ordinal properties of attributes, the dominance-based rough set theory can be used to solve multi-attribute auction problems. There have been many successful applications of the dominance-based rough set theory to multi-attribute decision fields.

3. Multi-attribute decision model by DRSA

3.1. Information system and the composition of the Pairwise Comparison Table (PCT)

A multi-attribute decision problem, which has a finite set of alternatives $B = \{x_1, x_2, \ldots, x_n\}$, can be treated as an information system. Generally, an information system is denoted as $S = (U, A, V, f)$, where U is a domain of discourse, so that the set of alternatives is $B \subset U$; A is the set of attributes; $V = \bigcup_{a \in A} V_a$, where V_a is the actual range of attribute a; and f is the information function, and $f : U \times A \to V$ makes $f(x, a) \in V_a$ for $\forall x \in U$, $a \in A$.

For $\forall a \in A$, T_a, denoted on set B, is a finite set of duality relations in light of attribute a, such that $\forall (x, y) \in B \times B$ uniquely ensures a dual relation $\tau \in T_a$. PCT is a defined data sheet for $S_{PCT} = (B \times B, A, T_a, g)$, where $B \times B$ is the set of pairwise comparisons; $T_A = \bigcup_{a \in A} T_a$; and $g : (B \times B) \times A \to T_A$ is an information function that yields $\forall (x, y) \in B \times B$, $a \in A$, $g[(x \times y), a] \in T_a$.

Assuming that the synthetic preference information of a small part of alternatives E ($E \subset B$) is known, and then PCT is a decision-making table, namely, $D_{PCT} = (E \times E, C \cup \{d\}, T_C \cup T_{\{d\}}, g)$, where E is a set of reference objects, and $E \times E$ is its set of pairwise comparisons, attribute set A consists of condition attribute set C and decision-making attribute $\{d\}$ of synthetic pair wise comparison, and $C \cap \{d\} = \Phi$, $C \cup \{d\} = A$; $T_C = \bigcup_{a \in C} T_a$, $T_a = \{P_a^h, h \in H_a\}$, p_a^h showing preference grades produced as alternatives in pairs for attribute a is compared, and H_a is a special set describing preference grades for attribute a.

For $\forall (x, y) \in E \times E$, $T_d = \begin{cases} S & x \succ y \\ S^C & otherwise \end{cases}$,

where '$\succ$' shows that x is at least as good as y; g is an information function and shows that $g[(x \times y), a \cup d] \in T_a \cup T_d$ for $\forall (x, y) \in E \times E$, $\forall a \in A$.

3.2. Determining the preference intension of multiple-grade dominance for attributes

Greco et al. (1999) put forward SPCT analysis by single grade dominance relation and assumed that the preference of all criteria has the same intension. This model is convenient for approximating and drawing decision rules, but is not precise enough. Taking different intensions of preference dominance relation into account for each criterion can yield a more accurate and real PCT (Wang et al. 2006).

The set of attributes $A = \{a_1, a_2, \ldots, a_n\}$ is where each attribute has its preference. Set A is divided into the set of condition attributes C and decision attribute $\{d\}$, such that $C \bigcap \{d\} = \Phi$, $C \bigcup \{d\} = A$, where the decision attribute is the synthetic valuation result of alternatives, reflecting the integrated preference relation of alternatives. Further, the decision attribute is quantitative and can be an index whose value is a cardinal number reflecting the integrated valuation of alternatives, or an ordinal number reflecting the ranking result of alternatives. A condition attribute that is a cardinal number shows a specific occurrence of the attribute for the alternative and should take the value in the actual range. Another form of condition attribute is the qualitative index of linguistic that describes the type.

The condition attributes of alternatives all have some preference relations, and different attributes have different preference intensions. Assume that the pair wise comparison of alternatives for each attribute can be shown by gradation preference P_a^h, namely, for $\forall a \in A$, $(x, y) \in E \times E$, and $h \in H_a$, $x P_a^h y$ shows that alternative x in light of attribute a dominates alternative y by the intension of h; when $h > 0$, it shows that alternative x dominates alternative y, when $h = 0$, it shows that alternative x does not vary against alternative y, and when $h < 0$, it shows that alternative x is dominated by alternative y. H_a is the set of preference intensions generated as alternatives that are compared in pairs, and is confirmed through the following process:

For information system $S = (U, A, V, f)$, first ensure an increasing preference function concerning attribute $a \in A$, $r_a : E \to R$. When an attribute is a numerical value, for the alternative $x \in E$, there is $r_a(x) = f(x, a)$; when an attribute is a linguistic description, the set of comments is V_a, there is $r_a(x) = k$, and $k \in \{1, 2, \ldots, |V_a|\}$. For example, if attribute a is a price, the values of the attributes for alternatives x_1, x_2, and x_3, respectively, are 20000, 18000, and 21000, namely, $f(x_1, a) = 20000$, $f(x_2, a) = 18000$, and $f(x_3, a) = 21000$, and then $r_a(x_1) = f(x_1, a) = 20000$, $r_a(x_2) = f(x_2, a) = 18000$, and $r_a(x_3) = f(x_3, a) = 21000$; if attribute a is the quality of a commodity,

$V_a = \{good, medium, bad\}$, $|V_a| = 3$, namely, there are three comments in the set of comments, and the attribute values of alternatives x_1, x_2, x_3 are good, medium, and bad, respectively. Specifically, $f(x_1, a) = 'medium'$, $f(x_2, a) = 'bad''$, and $f(x_3, a) = 'good'$. r_a is an increasing preference function, so the value of a function should increase with increasing preference, such that $r_a(x_1) = 2$, $r_a(x_2) = 1$, and $r_a(x_3) = 3$. Afterwards, the preference intension function is denoted as

$$h_a(r_a(x), r_a(y)) = \text{sgn}(r_a(x) - r_a(y)) \cdot \begin{cases} 0 & 0 \le |r_a(x) - r_a(y)| \le \Delta_{a1} \\ 1 & \Delta_{a1} < |r_a(x) - r_a(y)| \le \Delta_{a2} \\ \cdots \cdots \\ k & |r_a(x) - r_a(y)| > \Delta_{ak} \end{cases},$$

where Δ_{ak} is the threshold of the attribute properly given by the decision maker before the multi-attribute decision-making method. The Delphi method and analytic hierarchy process are the usual methods to decide on the threshold. The threshold can also be chosen according to industrial standards and national regulation. In the above example, since the attribute is price, and the threshold is $\Delta_{a1} = 2000$, then the preference grades of the pairwise comparisons of alternative are $h_a(r_a(x_1), r_a(x_1)) = 0$, $h_a(r_a(x_1), r_a(x_2)) = 1$, $h_a(r_a(x_1), r_a(x_3)) = 0$, and $h_a(r_a(x_2), r_a(x_1)) = -1$; therefore, $x_1 p_a^0 x_1$, $x_1 p_a^1 x_2$, $x_1 p_a^0 x_3$, and $x_2 p_a^{-1} x_1$ respectively show that according to attribute a, alternative x_1 is similar to alternative x_1, alternative x_1 dominates alternative x_2 with the intension of 1, alternative x_1 is similar to alternative x_3, and alternative x_2 does not dominate alternative x_1 with the intension of 1.

When the attribute is the quality of the commodity, and the thresholds are $\Delta_{a1} = 1$, $\Delta_{a2} = 2$, then the preference grades of the pairwise comparison of the alternative are $h_a(r_a(x_1), r_a(x_1)) = 0$, $h_a(r_a(x_1), r_a(x_2)) = 1$, $h_a(r_a(x_1), r_a(x_3)) = -1$, and $h_a(r_a(x_2), r_a(x_3)) = -2$; therefore, $x_1 p_a^0 x_1$, $x_1 p_a^1 x_2$, $x_1 p_a^{-1} x_3$, and $x_2 p_a^{-2} x_3$ respectively show that alternative x_1 is similar to alternative x_1, alternative x_1 dominates alternative x_2 with the intension of 1, alternative x_1 does not dominate alternative x_3 with the intension of 1, and alternative x_2 does not dominate alternative x_3 with the intension of 2 according to attribute a.

The above attributes discussed are the types of profit. As one attribute is the type of cost, it can be translated into the type of profit and then denoted as

preference intension grades, or denoted as the preference intension function as follows:

$$h_a(r_a(x), r_a(y)) = (-1) \cdot \operatorname{sgn}(r_a(x) - r_a(y)) \cdot \begin{cases} 0 & 0 \le |r_a(x) - r_a(y)| \le \Delta_{a1} \\ 1 & \Delta_{a1} < |r_a(x) - r_a(y)| \le \Delta_{a2} \\ \cdots\cdots \\ k & |r_a(x) - r_a(y)| > \Delta_{ak} \end{cases} .$$

3.3. Dominance relation based on PCT

For $\forall(x, y)$, $(w, z) \in E \times E$, in light of $a \in A$, $(x, y)D_a(w, z)$ shows that (x, y) dominates (w, z), or the intension that x dominates to y is at least the same as that when w dominates z. Then $\forall(x, y)$, $(w, z) \in E \times E$, $a \in A$, $h_a, k_a \in H_a, xP_a^h y$, and $wP_a^k z$ will yield $(x, y)D_a(w, z) \Leftrightarrow h_a \ge k_a$.

Based on D_a dominance relation, positive dominance $D_A^+(x, y)$ and negative dominance $D_A^-(x, y)$ can be introduced:

$$D_A^+(x, y) = \{(w, z) \in E \times E \mid (w, z)D_A(x, y)\} ,$$

$$D_A^-(x, y) = \{(w, z) \in E \times E \mid (x, y)D_A(w, z)\} .$$

The dual-dimension relation S and S^c defined on E about decision attribute $\{d\}$ can be considered. Regarding attribute set A, the upper approximation and the lower approximation of S are respectively defined by

$$\underline{C}(S) = \{(x, y) \in E \times E \mid D_A^+(x, y) \subseteq S\} ,$$

$$\overline{C}(S) = \{(x, y) \in E \times E \mid D_A^-(x, y) \bigcap S \neq \Phi\} = \bigcup_{(x, y) \in S} D_A^+(x, y) ,$$

while the upper approximation and the lower approximation of S^C are respectively defined as

$$\underline{C}(S^C) = \{(x, y) \in E \times E \mid D_A^-(x, y) \subseteq S^C\} ,$$

$$\overline{C}(S^C) = \{(x, y) \in E \times E \mid D_A^+(x, y) \bigcap S^c \neq \Phi\} = \bigcup_{(x, y) \in S^C} D_A^-(x, y) .$$

The following complementary properties come into existence: $\underline{C}(S) = E \times E - \overline{C}(S^C)$, $\overline{C}(S) = E \times E - \underline{C}(S^C)$, $\underline{C}(S^C) = E \times E - \overline{C}(S)$, and $\overline{C}(S^C) = E \times E - \underline{C}(S)$.

The boundaries of S and S^C are respectively denoted as follows:

$$bn(S) = \overline{P}(S) - \underline{P}(S), \quad bn(S^C) = \overline{P}(S^C) - \underline{P}(S^C), \text{ and } bn(S) = bn(S^C).$$

3.4. Acquiring the decision rules for multi-grade dominance

The upper accumulating preference and the lower accumulating preference are denoted as

$$\forall (x, y) \in E \times E, \ a \in A, \ h, k \in H_a$$

Such that when $k \geq h$ and $xP_a^k y$, then $xP_a^{\geq h} y$; when $k \leq h$, and $xP_a^k y$, then $xP_a^{\leq h} y$.

In terms of rough set approximation, the following three kinds of decision rules are derived from the appointed PCT:

$$P = \{a_1, a_2, \ldots, a_p\} \subseteq A, \ (h_{a1}, h_{a2}, \ldots, h_{ap}) \in H_{a1} \times H_{a2} \times \cdots \times H_{ap},$$

$D_{\geq}$ Decision rules: if $xP_{a1}^{\geq h_{a1}} y$ and $xP_{a2}^{\geq h_{a2}} y$, ..., and $xP_{ap}^{\geq h_{ap}} y$, then xSy, and supported by objects in pairs of $\underline{C}(S)$.

$D_{\leq}$ Decision rules: if $xP_{a1}^{\leq h_{a1}} y$ and $xP_{a2}^{\leq h_{a2}} y$, ..., and $xP_{ap}^{\leq h_{ap}} y$, then $xS^C y$, and supported by objects in pairs of $\underline{C}(S^C)$.

$D_{\geq \leq}$ Decision rules: if $xP_{a1}^{\geq h_{a1}} y$ and $xP_{a2}^{\geq h_{a2}} y$, ..., and $xP_{ak}^{\geq h_{ak}} y$, and $xP_{a(k+1)}^{\leq h_{a(k+1)}} y$ and $xP_{a(k+2)}^{\leq h_{a(k+2)}} y$, ..., and $xP_{ap}^{\leq h_{ap}} y$, then xSy or $xS^C y$, and supported by objects in pairs of $bn_c(S)$.

The decision rules above acquired on E are used for the whole set of alternatives B. Net flow value $S(x)$ is computed for each alternative $x \in B$ and can be used to rank and choose alternatives, where

$$S(x) = S^{++}(x) - S^{+-}(x) + S^{-+}(x) - S^{--}(x),$$

$S^{++}(x) = card(\{y \in B | \text{ at least one decision rule exists to supports } xS \ y\})$;

$S^{+-}(x) = card(\{y \in B | \text{ at least one decision rule exists to supports } yS \ x\})$;

$S^{-+}(x) = card(\{y \in B | \text{ at least one decision rule exists to supports } xS^C y\})$;

$S^{--}(x) = card(\{y \in B | \text{ at least one decision rule exists to supports } yS^C x\})$.

The alternative $x^* \in B$ satisfying $S(x^*) = \max_{x \in B} S(x)$ is the most optimal alternative.

4. Multi-attribute auction model by DRSA

4.1. Decision model

First, the qualification of suppliers for bidding is evaluated with the multi-attribute decision-making method. All suppliers that submitted bidding reports

are ranked, and the former m suppliers can be chosen to qualify for the later bidding. The attribute set of the required commodity is $C = \{a_1, a_2, \ldots, a_n\}$. The actual range of attribute a is denoted as V_a, and $V = \bigcup_{a \in C} V_a$. Thus, the whole bidding process is seen as the information system $S = (U, C, V, f)$, where U is the domain of discourse. The bidding alternative set B consists of the alternatives bided by m bidders each round, $B \subset U$; and f is an information function where $f : U \times C \to V$.

The attributes of the procured commodity are taken as condition attribute set C, assuming that there are no attributive redundancies. Decision attribute $\{d\}$ is obtained from the ranking result of m suppliers. This yields the comprehensive preference information of E, a small part of alternatives in the full information system. After confirming the preference grade set H_a of each condition attribute, PCT can be structured as $D_{PCT} = (E \times E, C \bigcup \{d\}, T_C \bigcup T_{\{d\}}, g)$, and this datasheet is a decision table. Moreover, the decision table derived through this method is rational and effective because it is based on the information of each bidder's bidding alternative, and it integrated the characters of commodity required by the buyer. The decision rules are derived in terms of DRSA from a small part of PCT for which the comprehensive preference information is known, and then are used on the bidders' alternatives in each round to obtain the decision value of pair wise comparison. Finally, the ranking of alternatives and the selection of the optimal alternative is implemented with scoring functions.

The bidding involves single goods and multiple attributes. Assume that there are R rounds of bidding, or that the bidding is constrained by time T. Before the r-th ($r \in \{1, 2, \ldots, R\}$) round begins, the procurer declares the bidding result of the previous round. Assume that the optimal bidding alternative of the r-th round is $b^{*(r)} = (a_1^{*(r)}, a_2^{*(r)}, \ldots, a_n^{*(r)})$, the worst being $b^{-(r)} = (a_1^{-(r)}, a_2^{-(r)}, \ldots, a_n^{-(r)})$. Based on the suppliers' actual bidding on $r-1$ round, the procurer declares the reference alternative $b_0^r = (a_{01}^r, a_{02}^r, \ldots, a_{0n}^r)$ and $b_0^r = b^{*(r-1)}$, which reflects his/her own bidding preference, with the worst alternative and preference grade thresholds Δ_{ak}^r for each condition attribute $a \in C$. Going along with the bidding, the procurer can modify Δ_{ak}^r according to the actual bid of each supplier.

Assume that there are m^r bidders on r round biding, and $m^r \leq m$. B^r is the bidding alternative set composed of m^r bidders' alternatives on r round. b_k^r is the bidding value of bidder k on the r-th round, and $b_k^r = (a_{k1}^r, a_{k2}^r, \ldots, a_{kn}^r)$, $S(b_k^r)$ is the net flow value, then
$$S(b_k^r) = S^{++}(b_k^r) - S^{+-}(b_k^r) + S^{-+}(b_k^r) - S^{--}(b_k^r),$$
Where

$$S^{++}(b_k^r) = card(\{b_j^r \in B^r \mid \text{at least one decision rule exixts to supports } b_k^r S b_j^r \});$$

$$S^{+-}(b_k^r) = card(\{b_j^r \in B^r \mid \text{at least one decision rule exixts to supports } b_j^r S b_k^r \});$$

$$S^{-+}(b_k^r) = card(\{b_j^r \in B^r \mid \text{at least one decision rule exixts to supports } b_j^r S^c b_k^r \});$$

$$S^{--}(b_k^r) = card(\{b_j^r \in B^r \mid \text{at least one decision rule exixts to supports } b_k^r S^c b_j^r \}).$$

To rank the bidding alternatives in set B^r for each round according to function $S(x)$, and to select the optimal alternative on this round $b^{*(r)} = (a_1^{*(r)}, a_2^{*(r)}, \dots a_n^{*(r)})$, making $S(b^{*(r)}) = \max\limits_{b^r \in B^r}\{S(b^r)\}$.

On the r-th round, the effective bid of bidder k is $b_k^r = (a_{k1}^r, a_{k2}^r, \dots, a_{kn}^r)$ and satisfies the following conditions:

(1) $\exists j \in \{1, 2, \dots, n\}$ making $a_{kj}^r > a_{0j}^r$;

(2) $card(M) \geq card(N)$, where $M = \{a_{kj}^r \mid a_{kj}^r > a_{kj}^{r-1}, j = 1, 2, \dots, n\}$ and $N = \{a_{kj}^r \mid a_{kj}^r < a_{kj}^{r-1}, j = 1, 2, \dots, n\}$.

If the bidder gives up bidding on any round, then there is the assumption that he/she will not participate in subsequent biddings.

4.2. The decision-making steps

The process of bidding based on the dominance relation rough multi-attribute decision-making model is as follows:

Step 1: For the supplier, perform a primary election with the multi-attribute decision theory regarding the bidding alternatives of the commodity to select m qualified suppliers for bidding. Use the ranking result of m suppliers as the decision attribute to construct PCT.

Step 2: Based on PCT reflecting the bidding information of suppliers, derive four kinds of dominance decision rules using DRSA.

Step 3: Before the r-th round begins, the buyer declares his/her preference alternative b_0^r and the preference grade thresholds Δ_{ak}^r of each condition attribute.

Step 4: Suppliers carry through the r-th round bidding. First, the decision maker judges the valid bid and changes the information of the bidding alternative into PCT, obtains the decision value of each alternative on the present round with dominance decision rules to rank all alternatives, and derives the optimal alternative $b^{*(r)}$ on the round.

Step 5: Continue the process until time T or the R round bid ends. The optimal alternative on the last round is the optimal procurement alternative, and its bidder is the winning bidder.

Step 6. End.

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5. Example

Assume that a firm needs to procure one commodity and has to select one supplier. The firm not only focuses on price but also on other faculties, such as capability for research and development, credit standing, servicing level, and financing status. The firm adopts a single-good and multi-attribute auction mode to choose the proper supplier. Information is publicized to invite public bidding, and then experts are invited to filter suppliers with the multi-attribute decision-making method to choose the qualified suppliers that should participate in bidding. The attribute set of commodity is

$C = \{$Price, Delivery Time, Ability of Study and Develpment,

Accounting Manner$\}$. Three suppliers are primarily filtered out to participate in the bidding and construct the decision table as shown in Table 1.

Table 1: Decision Table of the Qualified Suppliers

	Price ($)	Delivery Time (Month)	Ability of Study and Development	Accounting Manner	Ranking
x1	150	6	Good	Cash on Delivery	3
x2	175	5	Medium	Advance Payment	1
x3	135	4	Bad	Advance Payment	2

In Table 1, Supplier 2 has the highest price and is ranked as the first option since the buyer focuses on delivery time and scientific research level for the required commodity.

Table 2: The Standardized Decision Table and the Threshold of Attribute

	Price	Delivery Time	Ability of Study and Development	Accounting Manner	Ranking
x1	150	6	3	2	3
x2	176	5	2	1	1
x3	135	4	1	1	2
Δ_{a1}^{0}	20	0.9	0	0	
Δ_{a2}^{0}	40	1.9	1		

In Table 2, the linguistic attribute is changed into a quantitative attribute and price, the delivery time is the converse-attribute such as cost and capability for study and development, and the accounting manner is the positive attribute. The preference grade thresholds for each attribute are determined by experts for the pair wise comparisons of alternatives. The PCT, as shown in Table 3, is constructed.

Table 3: Pairwise Comparison Table

	Price	Delivery Time	Ability of Study And Development	Accounting Manner	Decision Result
(x1, x1)	0	0	0	0	S
(x1, x2)	1	-1	1	1	S^C
(x1, x3)	0	-2	2	1	S^C
(x2, x1)	-1	1	-1	-1	S
(x2, x2)	0	0	0	0	S
(x2, x3)	-2	-1	1	0	S
(x3, x1)	0	2	-2	-1	S
(x3, x2)	2	1	-1	0	S^C
(x3, x3)	0	0	0	0	S

Finally, the decision rules are extracted as follows when $\forall (x,\ y) \in B$:

$D_{\geq}$ Decision rules:

Rule 1 if $h_{a1} \geq 0$ and $h_{a2} \geq 0$, and $h_{a3} \geq 0$, and $h_{a4} \geq 0$, then xSy ;
Rule 2 if $h_{a1} \geq 0$ and $h_{a2} \geq 2$, then xSy ;

$D_{\leq}$ Decision rules:

Rule 3 if $h_{a1} \leq 0$ and $h_{a2} \leq -2$, then $xS^C y$;

$D_{\geq \leq}$ Decision rules:

Rule 4 if $h_{a1} \geq 1$, and $h_{a2} \leq -1$, and $h_{a4} \geq 1$, then xSy or $xS^C y$;
Rule 5 if $h_{a1} \leq -1$, and $h_{a2} \geq 1$, and $h_{a4} \leq -1$, then xSy or $xS^C y$;
Rule 6 if $h_{a1} \geq 1$, and $h_{a2} \leq 1$, and $\qquad\qquad$, then xSy or $xS^C y$;
Rule 7 if $h_{a1} \leq -1$, and $h_{a2} \geq -1$, and $h_{a3} \geq 1$, then xSy or $xS^C y$.

Before the first round begins, the buyer publicizes his/her satisfied bidding
$b^{*(0)} = (176, 5, \text{Medium}, \text{Advance Payment})$,

the worst bidding
$b^{-(0)} = (150, 6, \text{Good}, \text{Cash on Delivery})$,

and the preference grade thresholds of the alternative pairwise comparisons for each attribute in Table 2.

The third suppliers' bidding alternatives on the first round are indicated in Table 4. It also shows the results of the net flow values computed with the decision rules.

Table 4: The bidding alternatives on the 1 round, ranking result and thresholds of attribute

	Price	Delivery Time	Ability of Study And Development	Accounting Manner	Net Flow Value	Ranking
x1	147	5	3	2	2	1
x2	161	5	2	1	-4	3
x3	153	4	2	1	2	1
Δ_{a1}^0	5	0.5	0	0		
Δ_{a2}^0	10	1.5	1			

Before the second round of bidding, the buyer declares his/her most satisfied bidding alternatives

$b^{*(1)} = (147, 5, \text{Good}, \text{Cash on Delivery})$

$b^{*(1)} = (153, 4, \text{Medium}, \text{Advance Payment})$

and the worst bidding

$b^{-(1)} = (161, 5, \text{Medium}, \text{Advance Payment})$.

The preference grade thresholds of the alternative pairwise comparisons for each attribute are presented in Table 4.

Table 5 presents the third suppliers' bidding alternatives on the second round. The net flow values computed with the decision rules are also shown.

Table 5: The bidding alternatives on the 5 round and ranking result

	Price	Delivery Time	Ability of Study And Development	Accounting Manner	Net Flow Value	Ranking
x1	144	5	3	2	3	1
x2	153	5	2	2	-2	3
x3	138	4	2	1	0	2

If the bidding ends at this time, Supplier 1 is the winner, and the winning bidding is

$b^{*(2)} = b_1^2 = (144, 5, \text{Good}, \text{Cash on Delivery})$.

6. Conclusion

This paper proposes a multi-attribute decision method based on multi-grade DRSA to select the winner of a multi-attribute auction. Previously, bidding alternatives were evaluated using the fuzzy aggregative valuation method. An in-depth analysis of this model revealed that some of its characteristics, such as the reasoning process, are similar to those of a human mind. Further, the auction rules favor the declaration of the true preference information of the

seller and the buyer. This algorithm is simple and can easily be programmed for applications. The range of possible applications of the multi-attribute decision-making method and DRSA can be explored further. Future studies can analyze and design timely dynamic multi-attribute auction decision-making models based on the model developed in this paper.

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