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# A Novel Animal Migration Algorithm for Global Numerical Optimization

Qifang Luo[1], Mingzhi Ma[2], Yongquan Zhou[2]

[1] College of Information Science and Engineering,
Guangxi University for Nationalities, Nanning 530006, China
l.qf@163.com
[2] Key Laboratory of Guangxi High Schools Complex System and Computational Intelligence,
Nanning 530006, China
mamingzhi666@163.com, yongquanzhou@126.com

**Abstract.** Animal migration optimization (AMO) searches optimization solutions by migration process and updating process. In this paper, a novel migration process has been proposed to improve the exploration and exploitation ability of the animal migration optimization. Twenty-three typical benchmark test functions are applied to verify the effects of these improvements. The results show that the improved algorithm has faster convergence speed and higher convergence precision than the original animal migration optimization and other some intelligent optimization algorithms such as particle swarm optimization (PSO), cuckoo search (CS), firefly algorithm (FA), bat-inspired algorithm (BA) and artificial bee colony (ABC).

**Keywords:** animal migration optimization algorithms, exploration and exploitation, functions optimization

## 1. Introduction

In recent years, many research have been inspired by animal behavior phenomena for developing optimization techniques, such as firefly algorithm (FA) [1] in 2008, cuckoo search (CS) [2] in 2009, bat algorithm (BA) [2] in 2010, artificial bee colony (ABC) [3] in 2007, monkey algorithm (MA) [4] in 2008, frog-leaping algorithm (SFLA) [5], [6] in 2003. FA mimics flashes of fireflies attract mating partners or potential prey to find optima. CS simulates cuckoos choose a nest where the host bird just laid its own eggs and the first hatched cuckoo evicts the host eggs by blindly propelling the eggs out of the nest. BA simulates the behavior of bats' echolocation to find food or avoid obstacles by frequency and loudness. And ABC simulates natural bee colony foraging process to search and optimize the objectives by mutual cooperation. Because of its advantages of global, parallel efficiency, robustness and universality, these bio-inspired algorithms have been widely used in constrained optimization and engineering optimization[7], [8], [9], scientific computing, automatic control and other fields [10], [11], [12], [13], [14].

Animal migration behavior is found throughout the animal kingdom, the behavior stems from the change of natural environment, such as the change of feed, breeding, and climate, and then adaptive survival migration behavior. Animal migration usually has certain laws and routes, for instance, when an area is fall of temperature or lack of food, the animals will move to an area with mild climate and abundant food, this is a universal phenomenon exists in nature. The same birds, mammals, fish, reptiles, amphibians, insects and crustaceans all migrate, for example, the reindeer move southward to the taiga to escape from blizzard in winter, and move northward to tundra with rich food in spring. The wildebeest and zebra in Africa move to an area with rich aquatic plants when the rainy season comes, and return back to the usual habitats when the rainy season ends. The white whale, small stripe grain whale and other cetaceans often migrate by following the big fish as their food. The seal, fur seal and other sea mammal animals often climb back to a definite reproduction destination or an ice block to reproduce. In summary, animals like in food rich, water abundant, climatic conditions suitable area for survival, with time goes on, food and water reduce, climate change, living conditions change, the area can no longer provide the needs of the animals to survive, animal population will migrate to a new food and water rich, climatic conditions suitable area. Because of the long difficult journeys, individual animals will leave the migration of population in the process of animal population migration, and also some new individuals will join the migration of population in the process of animal population marching.

## 2. Related Work

Biogeography-based optimization is a recently proposed evolutionary algorithm, inspired by the migration behavior of island species [15]. Cuckoo search algorithm is inspired by the obligate brood parasitic behavior of some cuckoo species in combination with the Lévy flight behavior of some birds and fruit flies [16], and applied to classification problem [17], constrained global optimization [18], and various modification version cuckoo algorithm.

Bat Algorithm (BA) is a novel meta-heuristic optimization algorithm based on the echolocation behavior of micro-bats, which was proposed by Xin-she Yang in 2010 [2]. This algorithm is applied to different area. Pei-wei Tsai etc. proposed an improved EBA to solve numerical optimization problems [19]; A multi-objective bat algorithm (MOBA) is proposed by Yang [20], which is first validated against a subset of test functions, and applied to solve multi-objective design problems such as welded beam design. In 2012, BA to solve the Brushless DC wheel motor problem [21].

Animal migration optimization (AMO) algorithm is a novel bio-inspired optimization algorithm by simulating animal migration behavior that proposed by X. Li and J. Zhang in 2013 [22], and applied to clustering analysis [23]. AMO simulates the widespread migration phenomenon in the animal kingdom, through the change of position, replacement of individual, and finding the optimal solution gradually. AMO has obtained good experimental results on many optimization problems.

These algorithms have been applied to various research areas and have gained a lot of success [24], [25], [26], [27], [28]. However, up to new, there is no algorithm that performs well in all the fields. Some algorithms perform much better for some particular problems, while worse for other problems. Until now, how to design a new heuristic algorithm for optimization problem is still open problem [29].

This paper presents a modified animal migration optimization (MAMO) algorithm. We proposed MAMO algorithm to improve the performance of AMO, this method guarantees the MAMO rapid convergence. By means of selecting the better solution space around the current solution, it will improve search ability and accelerate convergence speed, and it has obtained the global optima.

## 3. Animal Migration Optimization Algorithm (AMO)

Animal migration algorithm can be divided into animal migration process and animal updating process. In the migration process the algorithm simulates how the groups of animals move from current position to a new position. During this process, each individual should obey three main rules: (1) move in the same direction as its neighbors; (2) remain close to its neighbors; (3) avoid collisions with its neighbors. During the population updating process, the algorithm simulates how animals update by the probabilistic method.

### 3.1. Animal Migration Process

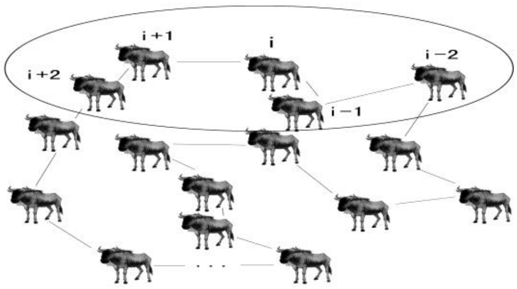



**Fig. 1** The concept of the neighborhood of an animal

During the animal migration process, an animal should obey three rules: (1) avoid collisions with your neighbors; (2) move in the same direction as your neighbors; and (3) remain close to your neighbors. In order to define concept of the local neighborhood of an individual, we use a topological ring, as has been illustrated in Fig. 1. For the sake of simplicity, we set the length of the neighborhood to be five for each dimension of the individual. Note that in our algorithm, the neighborhood topology is static and is defined on the set of indices of vectors. If the index of animal is $i$ then its neighborhood consists of animal having indices $i-2, i-1, i, i+1, i+2$, if the index of animal is 1, the neighborhood consists of animal having indices $NP-1, NP$, 1, 2, 3, etc. Once the neighborhood topology has been constructed, we select one neighbor randomly and update the position of the individual according to this neighbor, as can be seen in Formula (1):

$$X_{i,G+1} = X_{i,G} + \delta \cdot (X_{neiborhood,G} - X_{i,G}) \quad (1)$$

where $X_{neiborhood,G}$ is the current position of the neighborhood, $\delta$ is produced by using a random number generator controlled by a Gaussian distribution. $X_{i,G}$ is the current position of $i$ th individual, and $X_{i,G+1}$ is the new position of $i$ th individual.

### 3.2. Population Updating Process

During the population updating process, the algorithm simulates how some animals leave the group and some join in the new population. Individuals will be replaced by some new animals with a probability *Pa*. The probability is used according to the quality of the fitness. We sort fitness in descending order, so the probability of the individual with best fitness is 1/*NP*, the individual with worst fitness, by contrast, the probability is 1, and this process can be shown in Algorithm 1.

**For** $i=1$ to $NP$
  **For** $j=1$ to $D$
    **If** $rand > Pa$
      $X_{i,G+1} = X_{r1,G} + rand \cdot (X_{best,G} - X_{i,G}) + rand \cdot (X_{r2,G} - X_{i,G})$
    **End If**
  **End For**
**End For**

**Algorithm 1**. Population updating process

$r_1, r_2 \in [1,...,NP]$ are randomly chosen integers, and $r_1 \neq r_2 \neq i$ . After producing the new solution $X_{i,G+1}$ , it will be evaluated and compared with the $X_{i,G}$ , we choose the individual with a better objective fitness, as can be seen in Formula (2):

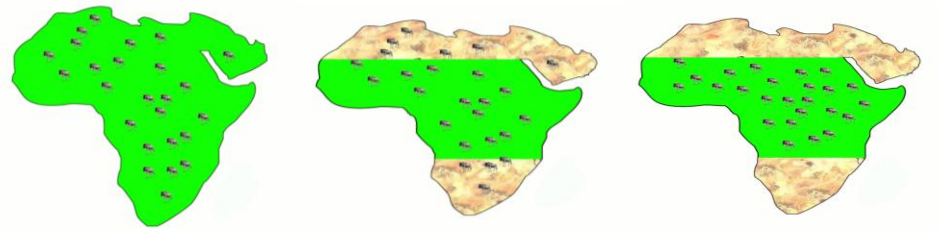
$$X_i = \begin{cases} X_{i,G} & if \ \ f(X_{i,G}) \, is \, better \, than \, f(X_{i,G+1}) \\ X_{i,G+1} & otherwise \end{cases} \tag{2}$$

To verify the performance of AMO, 23 benchmark functions were tested. The results show that the proposed algorithm clearly outperforms other evolution algorithms.

## 4. The Modified Animal Migration Process

In nature, animals to survive, all animals migrate toward the place with enough food. In this paper, the place is namely living area, and migrating animals have a leader. The proposed modified AMO algorithm established a living area by the leader animal (the individuals with best fitness value) and animals migrate from current locations migrate into this new living area to simulate animal migration process.

At first, there are *NP* animals live in living area, as shown in Fig. 2 (a), movement, eating, drinking, reproduction and so on, some individuals move randomly, and its' position update, then we calculate the best position of animals by fitness function and record it. But the amount of food or water gradually diminished as the time wore on, as shown in Fig. 2 (b), some animals migrate from the current areas which have no food and water to a new area with abundant food and water, as shown in Fig. 2 (c). In Fig. 2, the green parts represent the living areas with abundant food and water, animals can live in these areas, and the yellow parts represent the areas that lack of food or water, animals can no longer live in these areas, they must migrate to a new living area (the green parts in Fig. 2 (c)). We shrink the living area after a period of time, as shown in Fig. 2 (a) and (c)), and then animals migrate to the new living area ceaselessly. Because the animals living area is smaller to smaller (by Formula (3) and (4)), after each iteration, the individuals get closer and closer to the best individual, so we can accelerate the convergence velocity and precision of the algorithm to some extent.



(a) The $G$ -*th* iteration living area (b) Begin to migrate (c) The $G+1$ -*th* iteration living area

**Fig. 2** Animals migration process

The boundary of the living area is established by

$$low = X_{best} - R \, , up = X_{best} + R \tag{3}$$

$$R = \rho \cdot R \tag{4}$$

where $X_{best}$ is the leader animal, $low$ and $up$ are the lower and upper bound of the living area, $R$ is living area radius, $\rho$ is shrinkage coefficient, $\rho \in (0,1)$, $low$, $up$ and $R$ are all $1 \times D$ row vector.

In general, the original value of $R$ depends on the size of the search space. As iterations goes on, a big value of $R$ improves the exploration ability of the algorithm and, a small value of $R$ improves the exploitation ability of the algorithm.

## 5. The Modified Animal Migration Optimization Algorithm (MAMO)

The basic framework of the MAMO algorithm including: (1) animals live in living area; (2) as time goes on, food and water rations are reduced, hence, animals migrate to food rich, water sufficient, climate conditions suitable area; (3) individuals of animal population update; (4) animals live in new living area. So in modified animal migration optimization, mainly includes: living process, migration process, populations updating process.

### 5.1. Initializing the Population

During the initialization process, The algorithm begins by initializing a set of $NP$ animal positions $X_1, X_2, X_3, ..., X_{NP}$ ,each animal position $X_i$ is a $D$ -dimensional vector containing parameter values to be optimized, such values are randomly and uniformly distributed between the pre-specified lower initial parameter bound $a_j$ and the upper initial parameter bound $b_j$. So the $j$ th component of the $i$ th vector as Formula (5):

$$x_{i,j} = a_j + rand_{i,j}[0,1] \cdot (b_j - a_j) \text{ , } i = 1,..., NP \text{ , } j = 1,..., D \tag{5}$$

where $rand_{i,j}[0,1]$ is a uniformly distribution random number between 0 and 1.

### 5.2. Animals Migration

During the migration process, because animals hunting, foraging or drinking in the living area, some parts of the living area are lack of food or water or climate condition change, some animals migrate from the current living area to the new area which has abundant food and water or climate condition suitable for living. We assume that there is only one living areas, animals out of the new living area would be generated randomly in the new living area, as depicted in Section 3.

### 5.3. Animals Live Area

During the living process, algorithm simulates individuals' positions randomly change in living area. Following the biological model, animals hunting, foraging or drinking in habitat, their positions randomly change, an individual move to a new position according to the current position of its neighborhoods, such behavioral rule is move randomly in living area and implemented considering by Formula (1) in Section 2.1.

### 5.4. Population Updating

During the population updating process, algorithm simulates some individuals get lost, some animals are preyed by their enemies or some animals leave the group and some join in the group from other groups or some new animals are born. In MAMO, we assume that the number of available animals is fixed, and every animal will be replaced by *Pa,* as shown in Section 2.2. Specific implementation steps of the modified animals migration optimization algorithm can be shown as follows:

```
Begin
Set the generation counter $G=0$, living area radius $R$,
  shrinkage coefficient $\rho$, and randomly initialize with a
  population of $NP$ animals $X_i$ in solution space
Evaluate the fitness for each individual $X_i$, record the best
  individual $X_{best}$
While stopping criteria is not satisfied do
    Establish a new living area by $low = X_{best} - R$, $up = X_{best} + R$
    Animals migrate into the new living area
    For $i$ = 1 to $NP$ do
        For $j$ = 1 to $D$ do
            $X_{i,G+1} = X_{i,G} + \delta \cdot \left(X_{neighborhood,G} - X_{i,G}\right)$
        End For
    End For
    For i=1 to $NP$ do
        Evaluate the offspring $X_{i,G+1}$
        If $X_{i,G+1}$ is better than $X_{i,G}$ then
            $X_i = X_{i,G+1}$
        End If
    End For
    For $i$ = 1 to $NP$
        For $j$ = 1 to $D$
            Select randomly $r1 \neq r2 \neq i$
            If rand > $Pa$ then
                $X_{i,G+1} = X_{r1,G} + rand \cdot (X_{best,G} - X_{i,G}) + rand \cdot (X_{r2,G} - X_{i,G})$
            End If
        End For
    End For
    For $i$ = 1 to $NP$ do
        Evaluate the offspring $X_{i,G+1}$
        If $X_{i,G+1}$ is better than $X_i$ then
            $X_i = X_{i,G+1}$
        End If
    End For
    Memorize the best solution achieved so far
    $R = R \cdot \rho$
End while
End
```

**Algorithm 2**. A modified animal migration optimization algorithm (MAMO)

## 6. Experiments Result and Analysis

In this section, the 23 benchmark test functions have been used to test the performance of the proposed MAMO algorithm. The test functions are classified into 3 different categories:

(1) Unimodal high-dimensional test functions : $f_{01} - f_{07}$.

(2) Multimodal high-dimensional test functions: $f_{08} - f_{13}$.

(3) Multimodal low-dimensional test functions: $f_{14} - f_{23}$

**Table 1**. Benchmark functions

| Benchmark test functions | D | Range | Optimum |
|---|---|---|---|
| $f_{01}(x) = \sum_{i=1}^{n} x_i^2$ | 30 | [-100,100] | 0 |
| $f_{02}(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$ | 30 | [-10,10] | 0 |
| $f_{03}(x) = \sum_{i=1}^{n} (\sum_{j=1}^{i} x_j)^2$ | 30 | [-100,100] | 0 |
| $f_{04}(x) = \max \lvert x_i \rvert, 1 \le i \le n$ | 30 | [-100,100] | 0 |
| $f_{05}(x) = \sum_{i=1}^{n} [100(x_{i+1} - x_i^2)^2 + (1 - x_i^2)^2]$ | 30 | [-30,30] | 0 |
| $f_{06}(x) = \sum_{i=1}^{n} \lfloor x_i + 0.5 \rfloor$ | 30 | [-100,100] | 0 |
| $f_{07}(x) = \sum_{i=1}^{n} i \cdot x_i^4 + random[0.1)$ | 30 | [-1.28,1.28] | 0 |
| $f_{08}(x) = \sum_{i=1}^{n} -x_i \cdot \sin(\sqrt{\lvert x_i \rvert})$ | 30 | [-500,500] | -418.9829**n* |
| $f_{09}(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos 2\pi x_i + 10]$ | 30 | [-5.12,5.12] | 0 |
| $f_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos 2\pi x_i\right) + 20 + e$ | 30 | [-32,32] | 0 |
| $f_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}}) + 1$ | 30 | [-600,600] | 0 |
| $f_{12}(x) = \frac{\pi}{D}\left\{10\sin^2(\pi y_1) + \sum_{i=1}^{D-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right] + (y_D - 1)^2\right\} + \sum_{i=1}^{D} u(x_i, 10, 100, 4)$ <br> $y_i = 1 + \frac{x_i + 1}{4}$ <br> $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a \le x_i \le a \\ k(-x_i - z)^m & x_i < a \end{cases}$ | 30 | [-50,50] | 0 |
| $f_{13}(x) = 0.1\left\{10\sin^2(\pi y_1) + \sum_{i=1}^{D-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right] + (y_D - 1)^2\right\} + \sum_{i=1}^{D} u(x_i, 10, 100, 4)$ | 30 | [-50,50] | 0 |

| | | | |
|---|---|---|---|
| $f_{14}(x) = \left[ \frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2} (x_i - a_{ii})^6} \right]^{-1}$ | 2 | [-65.53, 65.53] | 0.998004 |
| $f_{15}(x) = \sum_{i=1}^{11} \left[ a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4} \right]^2$ | 4 | [-5,5] | 0.0003075 |
| $f_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$ | 2 | [-5,5] | -1.0316285 |
| $f_{17}(x) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10(1 - \frac{1}{8\pi})\cos x_1 + 10$ | 2 | [-5,10]* [0,15] | 0.398 |
| $f_{18}(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$ | 2 | [-5,5] | 3 |
| $f_{19}(x) = -\sum_{i=1}^{4} c_i \exp\left[ \sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2 \right]$ | 3 | [0,1] | -3.8628 |
| $f_{20}(x) = -\sum_{i=1}^{4} c_i \exp\left[ \sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2 \right]$ | 6 | [0,1] | -3.3224 |
| $f_{21}(x) = -\sum_{i=1}^{5} \left[ (X - a_i)(X - a_i)^T + c_i \right]^{-1}$ | 4 | [0,10] | -10.1532 |
| $f_{22}(x) = -\sum_{i=1}^{7} \left[ (X - a_i)(X - a_i)^T + c_i \right]^{-1}$ | 4 | [0,10] | -10.4029 |
| $f_{23}(x) = -\sum_{i=1}^{10} \left[ (X - a_i)(X - a_i)^T + c_i \right]^{-1}$ | 4 | [0,10] | -10.5364 |

The mean and standard deviation results of 25 independent runs for each algorithm have been summarized in Tables 2, 3 and 4.

### 6.1. Experimental Setup

All of the algorithms were programmed in MATLAB R2008a, numerical experiment was set up on AMD Athlon(tm)II *4 640 processor and 2GB memory.

### 6.2. Parameters Setting

We initialize $D$ -dimensional row vector $R = \rho \cdot (b_j - a_j)$, $j \in [1, 2, ..., D]$, $a_j$ and $b_j$ are the lower bound and upper bound of the solution space. the population size $NP$ is 50. The maximum numbers of iterations are 1500 for $f_{01}, f_{06}, f_{10}, f_{12}$ and $f13$, 2000 for $f_{02}$ and $f_{11}$, 3000 for $f_{07}, f_{08}$ and $f_{09}$, 5000 for $f_{03}, f_{04}$ and $f_{05}$, 400 for $f_{15}$, 100 for $f_{14}, f_{16}, f_{17}, f_{19}, f_{21}, f_{22}$ and $f_{23}$, 30 for $f_{18}$, and 200 for $f_{20}$.

Different test functions have different iterations, shrinkage coefficient $\rho$ for all test functions should be changed according to the each number of iteration to get a living area with same size at the end of iteration. In order to set a unified model to define $\rho$ , Numerical experiments results showed that MAMO algorithm can achieve more ideal effect if we choose the parameter values $\rho$ = 0.99 for iteration *iter* =2000, and the final living area radius $R_{final}$ = $0.99^{2000}$ =1.8638E-09 (by Algorithm 2) which is small enough to gather the individuals into the living area. We set this $R_{final}$ as standard to set other $\rho$ . An empirical model has been developed by following equation:

$$\rho = 0.99^{\frac{2000}{iter}} \tag{6}$$

where $\rho$ is shrinkage coefficient, *iter* is the number of iteration.

### 6.3. Comparison of MAMO with PSO, CS, FA, BA, ABS, and AMO

To demonstrate that the MAMO algorithm's performance, we compared MAMO with FA [1], CS [2], BA [3], ABC [4], AMO [22], PSO [28], respectively using the best, worst, mean and standard deviation value to compare their performance. The setting values of algorithm control parameters of the mentioned algorithms are given below.

FA: according to [1], $\alpha_0 = 0.5$ , $\beta_0 = 0.2$ and $\gamma = 1.0$ , the population size is 100.

CS: according to [2], $\beta = 1.5$ , $Pa = 0.25$ , the population size is 50, because of this algorithm has two phases.

BA: according to [3], loudness $A = 0.25$ , pulse rate $r = 0.5$ , $\alpha = 0.95$ , $\gamma = 0.5$ , the population size is 50, because of this algorithm has two phases.

ABC: according to [4], $\mathrm{limt} = 5D$ , the population size is 50, because of this algorithm has two phases.

AMO: according to [22], the population size is 50, because of this algorithm has two phases.

PSO: according to [28], weight factor $\omega$ linear decrease from 0.9 to 0.4, $c_1 = c_2 = 1.49445$ . The population size is 100.

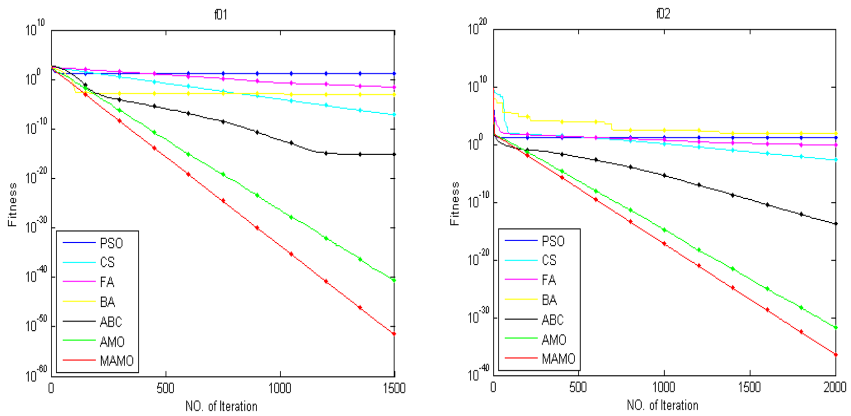
**Unimodal High-dimensional Functions Test Results and Analysis.** In the experiments, the mean results of 25 independent runs for $f_{01} - f_{07}$ are listed in Table 2. Functions $f_{01} - f_{07}$ are minimum problems, the convergence rate of search algorithm is more important for unimodal function than the final results. The best value, the worst value, mean value and the standard deviation value (Std) are recorded in Table 2 to 4. Generally, we use the index Std measure the performance of the algorithm.

**Table 2.** Experiment results of benchmark functions $f01 - f07$ for different algorithms

| Functions | | PSO | CS | FA | BA | ABC | AMO | MAMO |
|---|---|---|---|---|---|---|---|---|
| $f_{01}$ | Best | 0.0624 | 2.8652E-08 | 0.0191 | 8.4804E-04 | 5.9638E-16 | 9.0170E-42 | 7.3729E-54 |
| | Worst | 38.2278 | 1.4278E-07 | 0.0382 | 0.0014 | 7.7049E-16 | 2.2853E-40 | 2.9655E-51 |
| | Mean | 13.0262 | 7.0670E-08 | 0.0261 | 0.0011 | 7.0846E-16 | 6.7408E-41 | **2.9896E-52** |
| | Std | 13.7716 | 3.4388E-08 | 0.0050 | 1.1911E-04 | 4.4759E-17 | 5.9947E-41 | **6.2616E-52** |
| | **Rank** | **7** | **4** | **6** | **5** | **3** | **2** | **1** |
| $f_{02}$ | Best | 3.8578 | 4.3873E-04 | 0.5810 | 0.3792 | 5.6790E-15 | 4.0383E-33 | 1.4117E-37 |
| | Worst | 26.7085 | 0.0040 | 0.8523 | 226.6767 | 2.6254E-14 | 2.1191E-32 | 1.2250E-36 |
| | Mean | 14.2709 | 0.0020 | 0.6951 | 82.5828 | 1.6987E-14 | 1.0470E-32 | **4.3153E-37** |
| | Std | 7.8466 | 0.0008 | 0.0816 | 75.5580 | 5.5061E-15 | 5.0989E-33 | **3.2046E-37** |
| | **Rank** | **6** | **4** | **5** | **7** | **3** | **2** | **1** |
| $f_{03}$ | Best | 0.8527 | 5.6861E-04 | 1.4882 | 0.0016 | 1585.0790 | 8.3322E-05 | 7.7566E-13 |
| | Worst | 4814.2524 | 0.0097 | 3.9049 | 0.0034 | 4953.2237 | 0.0014 | 2.4874E-12 |
| | Mean | 2162.9249 | 0.0019 | 2.7936 | 0.0023 | 3212.6733 | 4.9669E-04 | **1.6245E-12** |
| | Std | 2043.5511 | 0.0019 | 0.5359 | 0.0004 | 851.0100 | 3.8937E-04 | **4.1675E-13** |
| | **Rank** | **7** | **4** | **5** | **3** | **6** | **2** | **1** |
| $f_{04}$ | Best | 0.2846 | 0.0022 | 0.2861 | 9.0079 | 10.5251 | 6.6727E-06 | 1.7786E-47 |
| | Worst | 20.8728 | 0.1305 | 0.3577 | 37.7763 | 24.8377 | 1.2931E-04 | 1.7475E-43 |
| | Mean | 10.0156 | 0.0533 | 0.3281 | 20.6236 | 17.5951 | 3.0408E-05 | **2.0915E-44** |
| | Std | 7.2464 | 0.0411 | 0.0178 | 8.3156 | 4.2192 | 2.6094E-05 | **4.5596E-44** |
| | **Rank** | **6** | **4** | **3** | **7** | **5** | **2** | **1** |
| $f_{05}$ | Best | 5.1569E+04 | 5.0827 | 33.7130 | 22.3030 | 0.0009 | 0.6576 | 2.5076 |
| | Worst | 9.3616E+05 | 11.8798 | 116.1006 | 29.4389 | 1.1830 | 11.1951 | 31.7648 |
| | Mean | 2.3551E+05 | 8.0953 | 43.7220 | 24.9738 | **0.1229** | 3.0220 | 11.2995 |
| | Std | 2.6421E+05 | 1.5060 | 22.8373 | 2.0811 | **0.2638** | 3.1375 | 5.5476 |
| | **Rank** | **1** | **3** | **7** | **5** | **2** | **4** | **6** |
| $f_{06}$ | Best | 618 | **0** | **0** | 11604 | **0** | **0** | **0** |
| | Worst | 1181 | **0** | **0** | 35776 | **0** | **0** | **0** |
| | Mean | 892 | **0** | **0** | 24728 | **0** | **0** | **0** |
| | Std | 214 | **0** | **0** | 6704 | **0** | **0** | **0** |
| | **Rank** | **2** | **1** | **3** | **1** | **1** | **1** | **1** |
| $f_{07}$ | Best | 8.1299 | 0.0050 | 0.0066 | 0.0009 | 0.0973 | 0.0201 | 0.0010 |
| | Worst | 37.6529 | 0.0183 | 0.0788 | 0.0047 | 0.1815 | 4.1672 | 0.0057 |
| | Mean | 21.7959 | 0.0091 | 0.0202 | 0.0026 | 0.1357 | 0.5973 | **0.0023** |
| | Std | 9.7284 | 0.0028 | 0.0160 | **0.0009** | 0.0245 | 1.3479 | 0.0011 |
| | **Rank** | **7** | **5** | **3** | **1** | **4** | **6** | **2** |

From Table 2, we can conclude that the results (Rank) obtained by MAMO are clearly better than the other algorithms for all 7 test functions except $f_{05}$; AMO is a little better than MAMO; the PSO is the worst for most functions. For $f_{01}$, the mean and standard deviation value of MAMO are 2.9896E-52 and 6.2616E-52 respectively which are 9 orders of magnitude better than AMO. The mean value of CS and ABC are 7.0670E-08 and 7.0846E-16. For $f_{02}$, the best solution that MAMO, AMO and ABC give are 1.4117E-37, 4.0383E-33 and 5.6790E-15, and the standard deviation value of MAMO is more better. For $f_{03}$ and $f_{04}$, MAMO provides the outstanding solution, and the means of MAMO are at least 8 and 39 orders of magnitude better than AMO and other algorithms. For $f_{05}$, the mean value of MAMO and AMO are 3.0220 and 11.2995 respectively, while ABC gives a better solution 0.1229. For $f_{07}$, The mean value of MAMO is 0.0023 which is better than other algorithms. Fig.3 (a-g) shows that the fitness function curve evolution of each algorithm for $f_{01}$ to $f_{07}$. These figures show that MAMO has faster convergence rate and the higher optimizing precision in solving multidimensional unimodal functions. Fig.4 (a-g) shows that the graphical analysis results of the ANOVA tests. As can be seen in Fig. 4, when solving function $f_{01}$ to $f_{07}$, most of the algorithms can obtain the stable optimal value after 25 iteration except PSO algorithm.

**Multimodal High-dimensional Functions Test Results and Analysis.** For multimodal functions $f_{08}$-$f_{13}$, in contrast to unimodal, have many local minima/maxima which are, in general, more difficult to optimize. The final results are more important because of this function can reflect the algorithm's ability to escape from local optima and obtain the global optimum. We have tested the experiments on $f_{08}$-$f_{13}$ where the number of local minima increases exponentially as long as the dimension of the function increases. Table 3 summarizes the best value, the worst value, mean value and the standard deviation value results of 25 independent runs for the selected functions. Generally, we use the index StdDev measure the performance of the algorithm.




(a) (b)

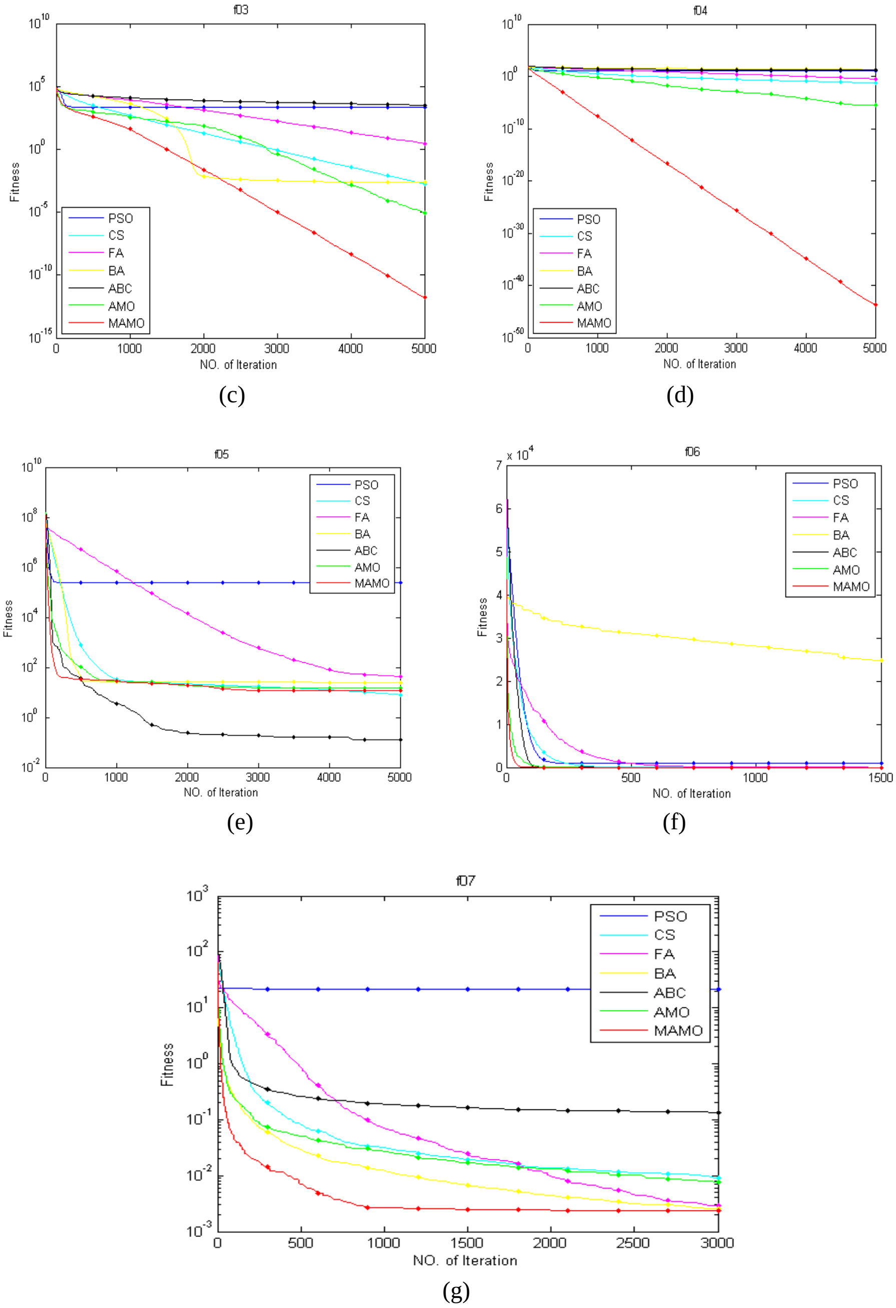
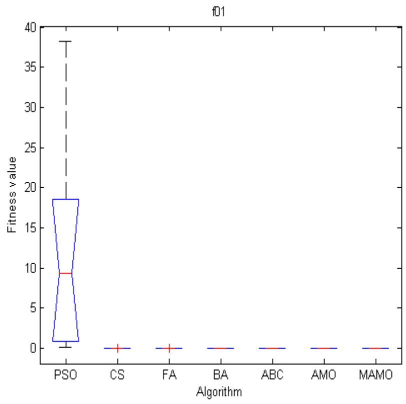
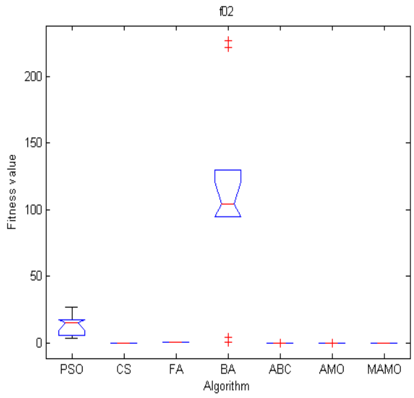


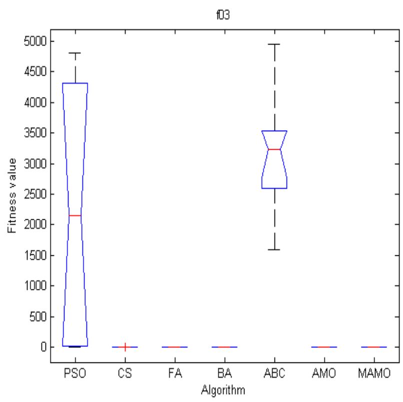

**Fig. 3.** Fitness function curve evolution for $f_{01}$-$f_{07}$


f01
Fitness value
PSO CS FA BA ABC AMO MAMO
Algorithm

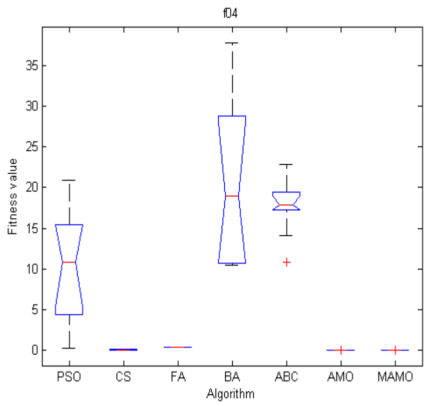
(a)


f02
Fitness value
PSO CS FA BA ABC AMO MAMO
Algorithm

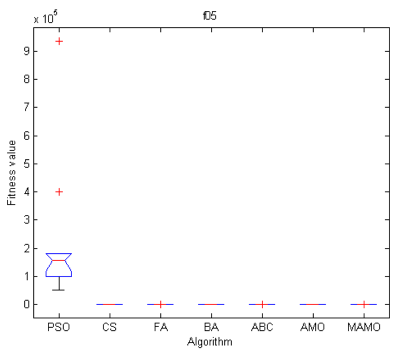
(b)


f03
Fitness value
PSO CS FA BA ABC AMO MAMO
Algorithm

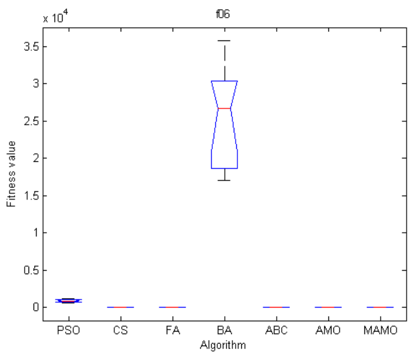
(c)


f04
Fitness value
PSO CS FA BA ABC AMO MAMO
Algorithm

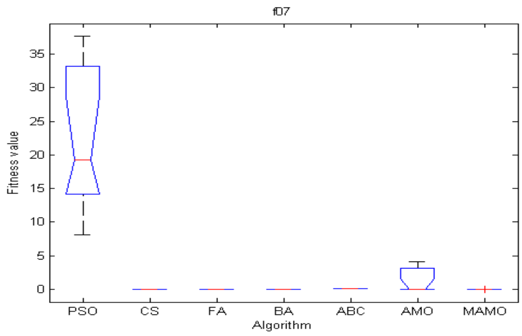
(d)


f05
x 10^5
Fitness value
PSO CS FA BA ABC AMO MAMO
Algorithm

(e)


f06
x 10^4
Fitness value
PSO CS FA BA ABC AMO MAMO
Algorithm

(f)




(g)

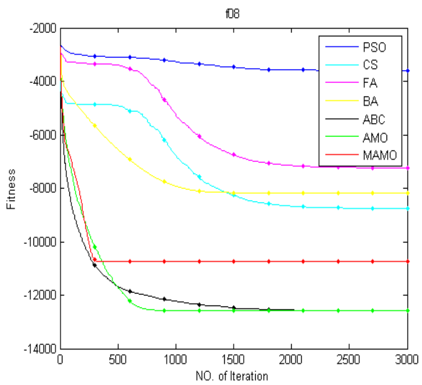
**Fig. 4.** ANOVA tests for $f_{01}$-$f_{07}$

**Table 3.** Experiment results of benchmark functions $f08 - f13$ for different algorithms

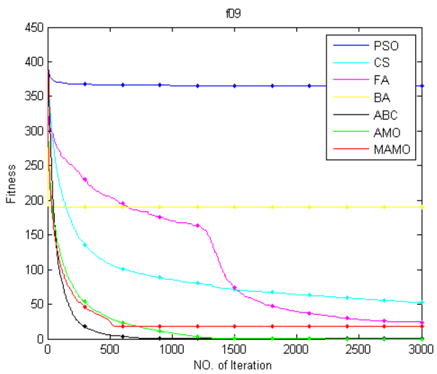
| Functions | | PSO | CS | FA | BA | ABC | AMO | MAMO |
|---|---|---|---|---|---|---|---|---|
| | Best | -6794.0585 | -9447.6445 | -7947.6445 | -9391.2125 | -12569.4866 | -12569.4866 | -11760.1553 |
| | Worst | -2753.3192 | -7433.8598 | -5933.8598 | -6587.5532 | -12569.4866 | -12569.4866 | -9430.7862 |
| $f_{08}$ | Mean | -3597.9022 | -8754.0246 | -7234.9052 | -8169.9329 | -12569.4866 | **-12569.4866** | -10755.3884 |
| | Std | 897.3297 | 591.0898 | 553.2876 | 761.6005 | 1.0668E-07 | **4.0674E-13** | 600.5346 |
| | **Rank** | **7** | **4** | **3** | **6** | **2** | **1** | **5** |
| | Best | 331.8060 | 34.4147 | 12.6914 | 131.4967 | 0 | 0 | 7.9597 |
| | Worst | 402.7944 | 67.2098 | 45.5377 | 232.0392 | 3.4106E-13 | 0 | 32.8336 |
| $f_{09}$ | Mean | 365.0584 | 52.2082 | 23.9947 | 190.2430 | 8.8107E-14 | **0** | 17.6108 |
| | Std | 17.1292 | 9.0984 | 6.4741 | 30.3222 | 8.1338E-14 | **0** | 5.5406 |
| | **Rank** | **6** | **5** | **4** | **7** | **2** | **1** | **3** |
| | Best | 16.7332 | 0.0263 | 0.3164 | 15.7627 | 2.9885E-10 | 4.4409E-15 | 4.4409E-15 |
| | Worst | 19.7878 | 1.3344 | 0.4961 | 19.2294 | 4.3636E-09 | 4.4409E-15 | 4.4409E-15 |
| $f_{10}$ | Mean | 18.2910 | 0.4648 | 0.4186 | 18.0883 | 1.9940E-09 | 4.4409E-15 | 4.4409E-15 |
| | Std | 0.7456 | 0.4337 | 0.0465 | 0.8667 | 1.1632E-09 | **0** | **0** |
| | **Rank** | **5** | **4** | **3** | **6** | **2** | **1** | **1** |
| | Best | 29.2946 | 9.34444E-09 | 0.8163 | 379.8683 | 0 | 0 | 0 |
| | Worst | 108.3321 | 2.14236E-07 | 0.9571 | 1954.0418 | 3.7526E-14 | 0 | 0 |
| $f_{11}$ | Mean | 57.2072 | 4.59672E-08 | 0.9008 | 1354.8913 | 2.9143E-15 | **0** | **0** |
| | Std | 15.6863 | 4.40214E-08 | 0.0379 | 492.8702 | 8.1411E-15 | **0** | **0** |
| | **Rank** | **5** | **3** | **4** | **6** | **2** | **1** | **1** |
| | Best | 1.3873E+07 | 0.0700 | 0.0014 | 0.0372 | 4.6085E-16 | 1.5705E-32 | 1.5705E-32 |
| $f_{12}$ | Worst | 1.9462E+08 | 1.0173 | 0.0028 | 0.9320 | 7.6901E-16 | 1.5705E-32 | 1.5705E-32 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean | 6.9715E+07 | 0.4865 | 0.0020 | 0.5112 | 6.5351E-16 | **1.5705E-32** | **1.5705E-32** |
| | Std | 5.5922E+07 | 0.2755 | 0.0003 | 0.2857 | 8.4555E-17 | **2.7369E-48** | **2.7369E-48** |
| | **Rank** | **3** | **5** | **4** | **6** | **2** | **1** | **1** |
| | Best | 5.5425E+07 | 0.0426 | 0.0012 | 0.0054 | 3.7274E-16 | 1.4998E-32 | 1.4998E-32 |
| | Worst | 3.4650E+08 | 1.1267 | 0.0023 | 1.2763 | 7.7667E-16 | 1.4998E-32 | 1.4998E-32 |
| $f_{13}$ | Mean | 1.8237E+08 | 0.4748 | 0.0019 | 0.6838 | 6.7482E-16 | **1.4998E-32** | **1.4998E-32** |
| | Std | 8.1652E+07 | 0.2814 | 0.0003 | 0.3803 | 1.1951E-16 | **5.4738E-48** | **5.4738E-48** |
| | **Rank** | **3** | **5** | **4** | **6** | **2** | **1** | 1 |

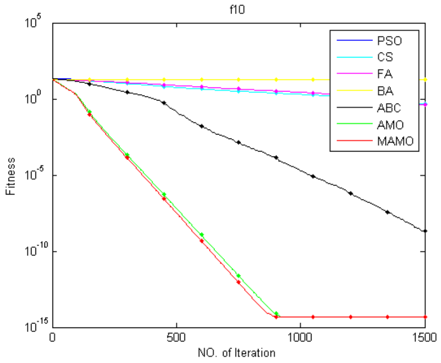
As can be seen rank line in Table 3, for test functions $f_{08} - f_{13}$, most of the solutions of MAMO and AMO get similar solution except $f_{08}$ and $f_{09}$. For $f_{08}$ and $f_{09}$, AMO achieves the optimal value. For $f_{10}$ to $f_{13}$, AMO and MAMO provide the same best mean and standard deviation value of all algorithms. For $f_{10}$, the best, worst, mean and standard deviation value of AMO and MAMO are all 0. Fig. 5 (a-f) shows the fitness function curve evolution. From Fig. 6, we can conclude that although MAMO and AMO converge to the same value, MAMO has a faster convergent rate than AMO when solving $f_{10}$, $f_{11}$, $f_{12}$ and $f_{13}$. Fig. 6 (a-f) shows the graphical analysis results of the ANOVA tests.



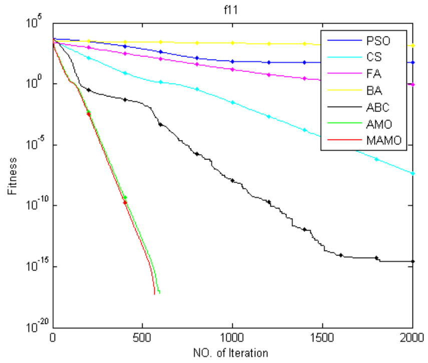

(a)



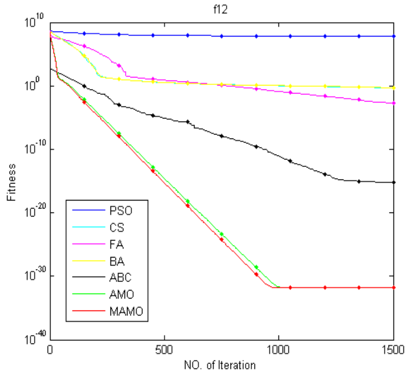

(b)



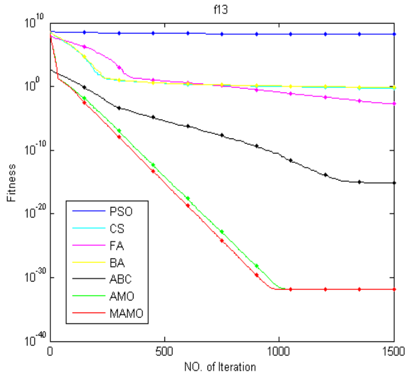

(c)




(d)

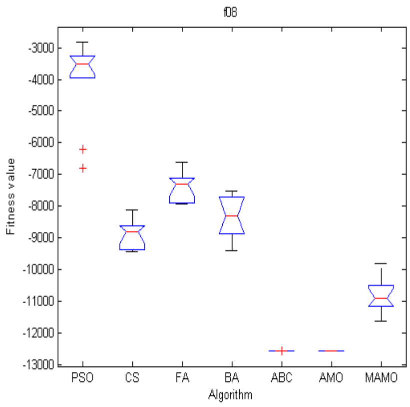



(e)

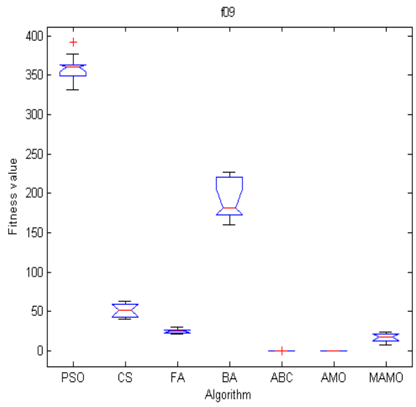



(f)

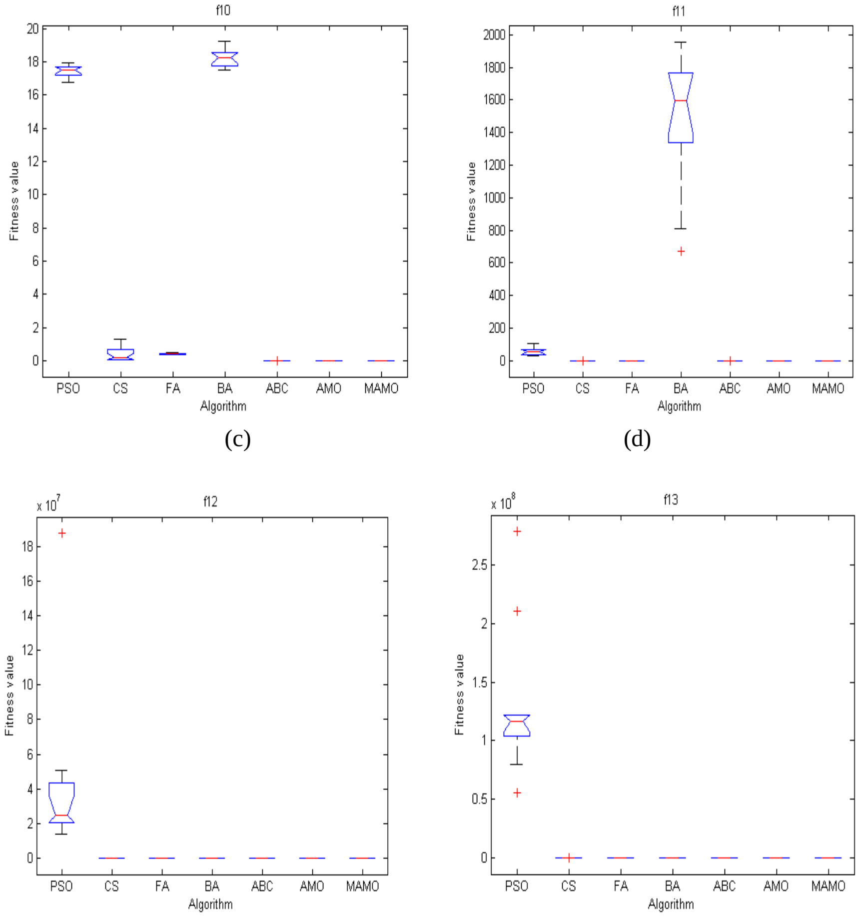
**Fig. 5.** Fitness function curve evolution for $f_{08}$-$f_{13}$




(a)




(b)




(c) (d)

(e) (f)

**Fig. 6.** ANOVA tests for $f_{08}$-$f_{13}$

**Multimodal Low-dimensional Functions Test Results and Analysis.** For multimodal low-dimensional functions $f_{14}$-$f_{23}$, they have only a few local minima and the dimension of the function is also small, so we use lower iterations to compare MAMO with other algorithms. In the experiment, the mean results of 25 independent runs are summarized in Table 4. Generally, we use the index StdDev measure the performance of the algorithm.

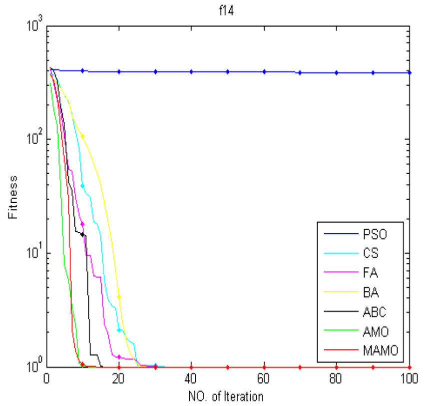
**Table 4.** Experiment results of benchmark functions $f_{14}$-$f_{23}$ for different algorithms

| Functions | | PSO | CS | FA | BA | ABC | AMO | MAMO |
|---|---|---|---|---|---|---|---|---|
| | Best | 0.99801 | 0.99800 | 0.99800 | 0.99800 | 0.99800 | 0.99800 | 0.99800 |
| $f_{14}$ | Worst | 499.80461 | 0.99800 | 0.99800 | 0.99800 | 0.99800 | 0.99800 | 0.99800 |
| | Mean | 382.97052 | **0.99800** | **0.99800** | **0.99800** | **0.99800** | **0.99800** | **0.99800** |

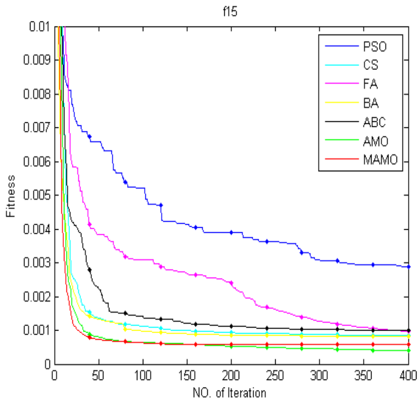
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Std | 189.27530 | 2.2067E-12 | 1.1764E-10 | 1.0066E-15 | 1.1827E-11 | 2.1642E-13 | **1.9860E-16** |
| | **Rank** | **7** | **4** | **6** | **2** | **5** | **3** | **1** |
| | Best | 0.00113 | 0.00052 | 0.00051 | 0.00031 | 0.00064 | 0.00032 | 0.00031 |
| | Worst | 0.00527 | 0.00105 | 0.00159 | 0.00159 | 0.00153 | 0.00049 | 0.00072 |
| $f_{15}$ | Mean | 0.00289 | 0.00084 | 0.00098 | 0.00081 | 0.00099 | **0.00041** | 0.00057 |
| | Std | 0.00127 | 0.00013 | 0.00030 | 0.00030 | 0.00020 | **4.5475E-05** | 0.00014 |
| | **Rank** | **6** | **2** | **5** | **5** | **4** | **1** | **3** |
| | Best | -1.03150 | -1.03163 | -1.03163 | -1.03163 | -1.03163 | -1.03163 | -1.03163 |
| | Worst | -1.02072 | -1.03163 | -1.02915 | -1.03163 | -1.03161 | -1.03163 | -1.03163 |
| $f_{16}$ | Mean | -1.02929 | **-1.03163** | -1.03148 | **-1.03163** | **-1.03163** | **-1.03163** | **-1.03163** |
| | Std | 0.00268 | 1.4305E-07 | 0.00053 | 1.1850E-07 | 3.8469E-06 | 1.4745E-11 | **2.9790E-16** |
| | **Rank** | **7** | **4** | **6** | **3** | **5** | **2** | **1** |
| | Best | 0.39899 | 0.39789 | 0.39789 | 0.39789 | 0.39789 | 0.39789 | 0.39789 |
| | Worst | 2.30267 | 0.39796 | 0.39798 | 0.39789 | 0.39790 | 0.39789 | 0.39789 |
| $f_{17}$ | Mean | 0.82386 | 0.39790 | 0.39791 | **0.39789** | 0.39789 | **0.39789** | **0.39789** |
| | Std | 0.51750 | 1.5795E-05 | 2.1140E-05 | 5.2758E-08 | 3.6487E-06 | **0** | **0** |
| | **Rank** | **6** | **4** | **5** | **2** | **3** | **1** | **1** |
| | Best | 3.08497 | 3.00977 | 3.00088 | 0.39789 | 3.02122 | 3.00100 | 3 |
| | Worst | 12.15882 | 5.63852 | 3.03221 | 0.39789 | 18.07006 | 3.03955 | 3 |
| $f_{18}$ | Mean | 5.18553 | 3.44519 | 3.01060 | 0.39789 | 5.37064 | 3.00902 | **3** |
| | Std | 2.24017 | 0.77291 | 0.00878 | 5.2758E-08 | 3.94782 | 0.01072 | **1.5132E-14** |
| | **Rank** | **6** | **5** | **3** | **2** | **7** | **4** | **1** |
| | Best | -3.85076 | -3.86278 | -3.86278 | -3.86278 | -3.86278 | -3.86278 | -3.86278 |
| | Worst | -3.02071 | -3.86267 | -3.86099 | -3.86274 | -3.86031 | -3.86278 | -3.86278 |
| $f_{19}$ | Mean | -3.60090 | -3.86277 | -3.86258 | -3.86277 | -3.86266 | **-3.86278** | **-3.86278** |
| | Std | 0.22133 | 2.3902E-05 | 0.00051 | 9.0408E-06 | 0.00054 | 1.6347E-15 | **1.9885E-16** |
| | **Rank** | **7** | **4** | **5** | **3** | **6** | **2** | **1** |
| | Best | -2.62396 | -3.32214 | -3.32234 | -3.32219 | -3.32237 | -3.32207 | -3.32237 |
| | Worst | -1.50458 | -3.31778 | -3.15394 | -3.20192 | -3.32235 | -3.32206 | -3.32237 |
| $f_{20}$ | Mean | -2.00192 | -3.32126 | -3.27488 | -3.25630 | -3.32237 | -3.32207 | **-3.32237** |
| | Std | 0.30426 | 0.00106 | 0.06519 | 0.05936 | 3.4784E-06 | 1.9844E-06 | **9.9711E-08** |
| | **Rank** | **5** | **4** | **7** | **6** | **3** | **2** | **1** |
| $f_{21}$ | Best | -5.05520 | -10.08303 | -10.15039 | -10.15303 | -10.15319 | -10.15320 | -10.15320 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Worst | -2.16927 | -9.24270 | -2.68018 | -2.63045 | -9.39489 | -8.27469 | -2.63047 |
| | Mean | -4.49265 | -9.79876 | -8.33948 | -5.90740 | **-10.04447** | -10.01521 | -8.40389 |
| | Std | 1.11378 | 0.23143 | 3.12530 | 3.53661 | **0.18117** | 0.41829 | 3.14839 |
| | **Rank** | **4** | **2** | **5** | **7** | **1** | **3** | **6** |
| | Best | -5.08767 | -10.37347 | -10.40096 | -10.40293 | -10.40287 | -10.40294 | -10.40294 |
| | Worst | -1.58721 | -9.51770 | -10.37968 | -2.75189 | -9.81857 | -10.09594 | -10.40294 |
| $f_{22}$ | Mean | -4.48409 | -9.97271 | -10.39251 | -5.48586 | -10.30291 | -10.38664 | **-10.40294** |
| | Std | 1.21181 | 0.24158 | 0.00615 | 2.61886 | 0.15940 | 0.06672 | **3.4628E-15** |
| | **Rank** | **6** | **5** | **2** | **7** | **4** | **3** | **1** |
| | Best | -5.12847 | -10.45131 | -10.53431 | -10.53604 | -10.53625 | -10.53641 | -10.53641 |
| | Worst | -2.85665 | -7.22276 | -10.49722 | -2.42733 | -9.67901 | -10.40656 | -10.53641 |
| $f_{23}$ | Mean | -4.57954 | -9.71234 | -10.52331 | -6.04331 | -10.33162 | -10.52719 | **-10.53641** |
| | Std | 0.73895 | 0.77320 | 0.01074 | 3.36106 | 0.24255 | 0.02931 | **9.9823E-16** |
| | **Rank** | **5** | **6** | **2** | **7** | **4** | **3** | **1** |

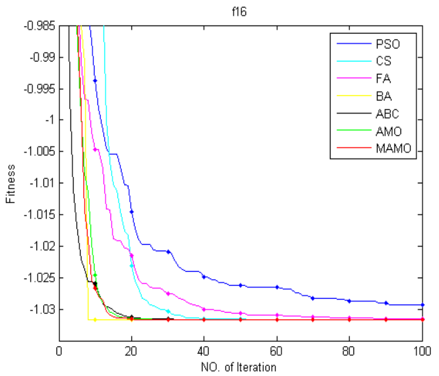
From rank line in Table 4, we can seen the best, worst, mean and standard deviation value of MAMO algorithm are superior when solving the multimodal low-dimensional functions. For $f_{14}$, results of all algorithms except PSO approximate to the optimal value 0.99800, while the standard deviations of MAMO is 1.9860E-16, which is better than 2.2067E-12, 1.1764E-10, 1.0066E-15, 1.1827E-11, 2.1642E-13 that obtained respectively by CS, FA, BA, ABC, AMO. For $f_{15}$, the most solutions of the algorithms are in the same order of magnitude, but the solution of AMO is more approximate to the optimum value. For polynomial functions $f_{16}$, $f_{17}$ and $f_{18}$, the mean of most of the algorithms achieve the same order of magnitude, but the mean value of MAMO are -1.03163, 0.39789 and 3 which all reached the globally optimal solution. For $f_{19}$ and $f_{20}$, the best, worst and the mean value are all -3.86278 and -3.32237 respectively, which embody the steady performance of MAMO. For $f_{21}$, $f_{22}$ and $f_{23}$, though the solution of MAMO is not satisfied when solving $f_{21}$, the mean values are significantly more approximate to the optimal value when solving $f_{22}$ and $f_{23}$, the best, worst and the mean value are the same and the standard deviation values are at least 13 orders of magnitude better than AMO and other algorithms. The optimization result of multimodal low-dimensional functions shows that the algorithm presented can solve premature convergence problem effectively and converge to the globally optimal solution. Fig. 7 (a-j) and Fig. 8 (a-j) show the fitness function curve evolution and the graphical analysis results of the ANOVA tests.


f14
Fitness
NO. of Iteration
PSO
CS
FA
BA
ABC
AMO
MAMO

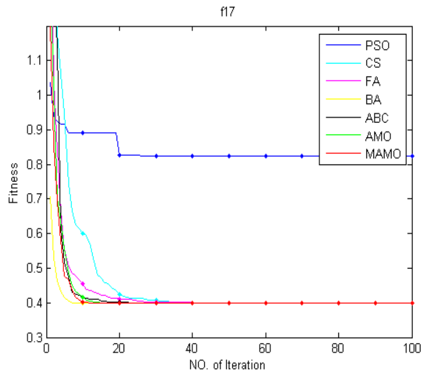
(a)


f15
Fitness
NO. of Iteration
PSO
CS
FA
BA
ABC
AMO
MAMO

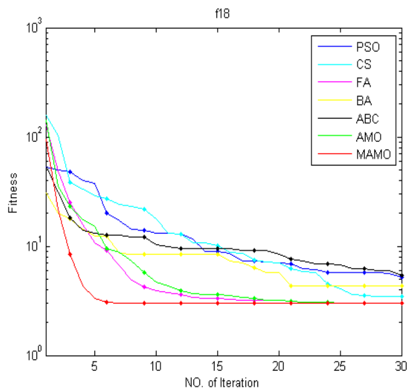
(b)


f16
Fitness
NO. of Iteration
PSO
CS
FA
BA
ABC
AMO
MAMO

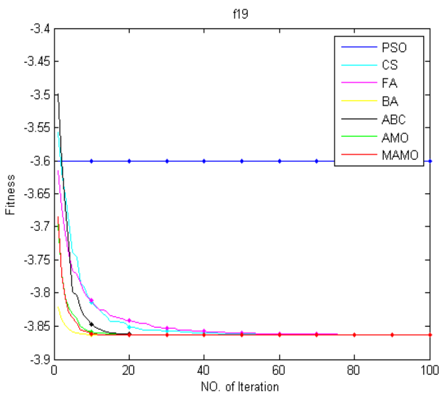
(c)


f17
Fitness
NO. of Iteration
PSO
CS
FA
BA
ABC
AMO
MAMO

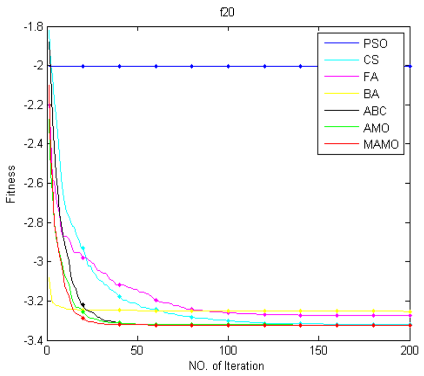
(d)


f18
Fitness
NO. of Iteration
PSO
CS
FA
BA
ABC
AMO
MAMO

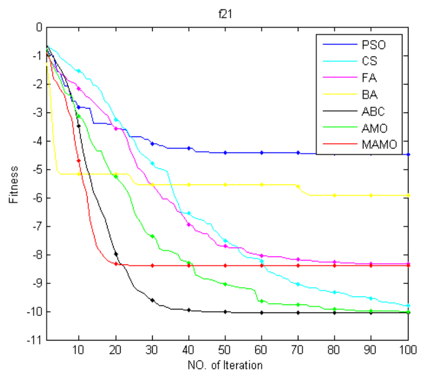
(e)


f19
Fitness
NO. of Iteration
PSO
CS
FA
BA
ABC
AMO
MAMO

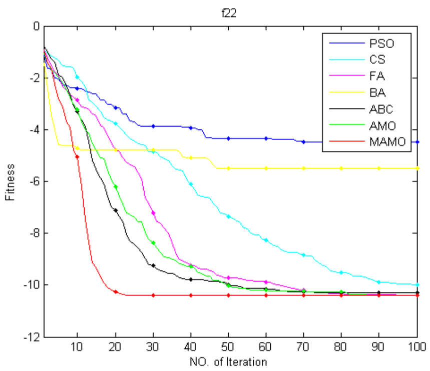
(f)



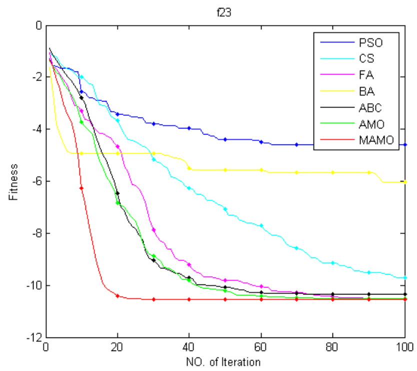

(g)




(h)

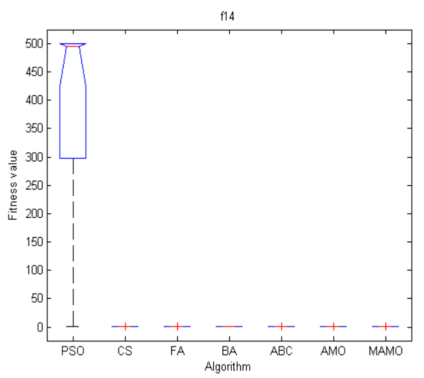



(i)

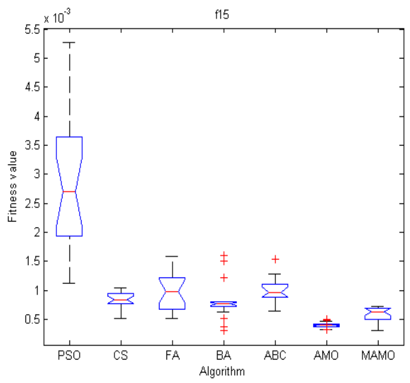



(j)

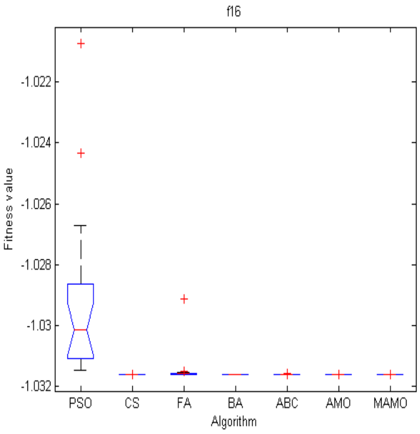
**Fig. 7.** Fitness function curve evolution for $f_{14}$-$f_{23}$



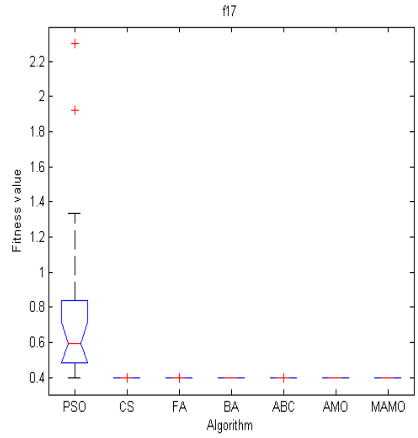

(a)



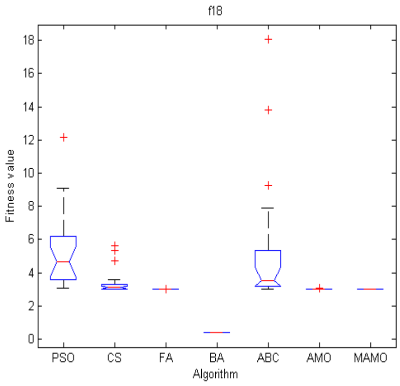

(b)


f16
Fitness value
Algorithm
PSO CS FA BA ABC AMO MAMO

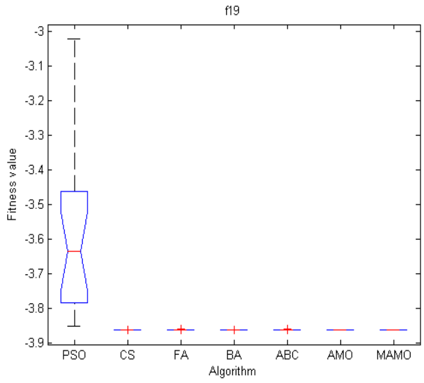
(c)


f17
Fitness value
Algorithm
PSO CS FA BA ABC AMO MAMO

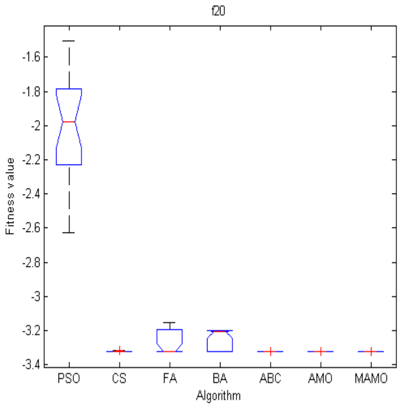
(d)


f18
Fitness value
Algorithm
PSO CS FA BA ABC AMO MAMO

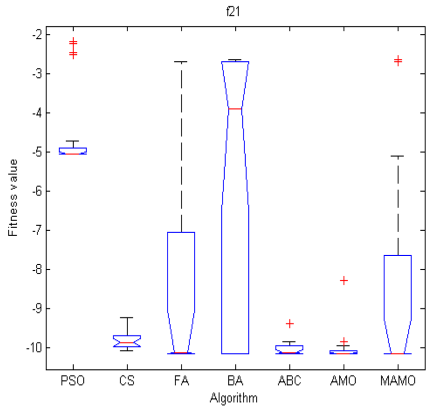
(e)


f19
Fitness value
Algorithm
PSO CS FA BA ABC AMO MAMO

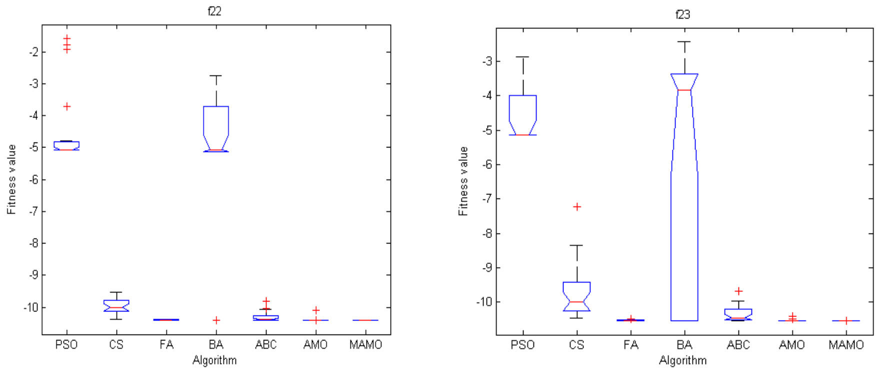
(f)


f20
Fitness value
Algorithm
PSO CS FA BA ABC AMO MAMO

(g)


f21
Fitness value
Algorithm
PSO CS FA BA ABC AMO MAMO

(h)



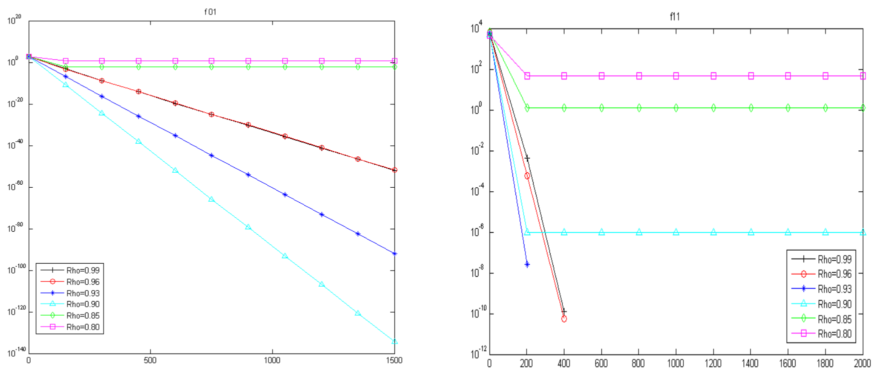

(i) (j)

**Fig. 8.** ANOVA tests for $f_{14}$-$f_{23}$

## 7. Evaluation of Living Area Radius

The performance and results of the proposed algorithms are greatly affected by the size of living area, a big value of $R$ can provide a big solution space, which could enhance the diversity of individuals, and a small value of $R$ can fast gather the individuals around the best individual. We adopted different shrinking coefficient $\rho$ to change the living area radius after each iteration, as shown in Formula (3) and (4), and set a unified model $\rho = 0.99^{\frac{2000}{iter}}$ in Formula (6). To study the extent of $R$ impacts on the proposed algorithm, we selected one unimodal high-dimensional function and one multimodal high-dimensional function separately, set different $\rho$ to evaluate the proposed algorithm.

Fig. 9 (a) shows the results of an experiment on $f_{01}$, we can conclude that if we choose $\rho = 0.90$, it has a better convergence precision than $\rho = 0.93$, $\rho = 0.96$, $\rho = 0.99$, while if we choose $\rho = 0.85$ and $\rho = 0.80$, MAMO algorithm plunges into local optima. So the best $\rho$ for $f_{01}$ must exist between 0.85 and 0.90. And likewise in Fig. 9 (b), for $f_{11}$, MAMO algorithm quickly converged at global optimum before 400 iterations if we choose $\rho = 0.99$, $\rho = 0.96$ or $\rho = 0.93$, while MAMO could not escape from poor local optima and to global optimum if we choose $\rho = 0.90$, $\rho = 0.85$ or $\rho = 0.80$. So the best $\rho$ for $f_{11}$ must exist between 0.90 and 0.93. The results suggest that a proper $\rho$ can greatly improve the algorithm convergence velocity and convergence precision, and an improper $\rho$ may lead the MAMO fall into local optimum. The evaluation experiment show that a big value of $R$ improves the exploration ability of the algorithm and a small value of $R$ improves the exploitation ability of the algorithm.




(a) (b)

**Fig. 9.** Fitness function curve evolution for $f_{01}$ and $f_{11}$ with different $\rho$

## 8. Conclusions

In this paper, to improve the deficiencies of the AMO algorithm, we modified the algorithm by using a new migration method. By 23 benchmark test functions, include unimodal high-dimensional, multimodal high-dimensional and multimodal Low-dimensional functions, we provide some comparisons of MAMO with PSO, CS, FA, BA, ABS, AMO and an experimental results show that MAMO algorithm has strong global searching ability and local optimization ability, MAMO has improved the convergence speed and convergence precision of AMO, therefore, it is very effective to solve complex functions optimization problems. At last, how to define a proper and unified radius of living area need to be considered in subsequent work.

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**Qifang Luo,** Associate Professor. She received the MS degree in computer science from Guangxi University in 2005, Her research interests are in the areas of computation intelligence, and intelligence information systems.

**Mingzhi Ma,** M.S., received the BS degree from College of Information Science and Engineering, Guangxi University for Nationalities, China, in 2015. His current research interest is in computation intelligence, swarm intelligence algorithm.

**Yongquan Zhou,** Ph.D & Prof, received the MS degree in computer science from Lanzhou University, Lanzhou, China, in 1993 and the PhD degree in computation intelligence from the Xiandian University, Xi’an, China, in 2006. He is currently a professor at Guangxi University for Nationalities. His research interests include computation intelligence, neural networks, and intelligence information processing.

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