A Fast Non-dominated Sorting Multi-objective Symbiotic Organism Search Algorithm for Energy Efficient Locomotion of Snake Robot*

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Abstract. This paper deals with energy efficient locomotion of a wheel-less snake robot. This is very crucial for potential applications of untethered snake robots. The optimum gait parameters for the energy efficient locomotion of the snake robot are obtained with two different multi-objective algorithms based on symbiotic organism search algorithm by considering both minimizing the average power consumption and maximizing the forward velocity of the robot. This paper also investigates the energy efficient locomotion of the snake robot under different environment conditions. The obtained results demonstrate that both proposed methods achieve satisfying stable results regarding power consumption reduction with optimal forward velocity for lateral undulation motion. However, it is seen that fast non-dominated sorting multi-objective symbiotic organism search algorithm provides advantage on obtaining a uniformly distributed solution set with a good diversity only in a single run. This paper is important in terms of presenting useful results for developing efficient motion and environmental adaptability of the snake robot.

Keywords: Energy efficiency, adaptive locomotion, friction condition, optimum gait parameters, symbiotic organism search algorithm, multi-objective optimization, snake robot

1. Introduction

Developed by inspiring from the perfect motions of biological snakes in unknown, irregular environments, snake robots give an outstanding locomotion in many challenging environments in comparison with other mobile robots such as legged, tracked and wheeled robots. This unique feature makes snake robots useful for many purposes in real-world applications such as firefighting, military purposes, rescue and search operations, maintenance of nuclear plants and pipelines. Energy efficient locomotion for especially such applications is very important for the snake robot. Despite this, many of studies focus on modelling, development, and control of these

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mechanism. There are very few studies addressing energy efficient locomotion for snake robot in the literature.

Powell’s method, a gradient-free optimization method, is used to maximize the forward velocity of the snake robot for a given environment by optimizing the parameters of the central pattern generator (CPG) [1]. In this study, only the amplitude and phase lag parameters of the motion pattern are handled, and the effect of frequency parameter on efficient motion is not examined. The experimental studies are executed for crawling motion on a horizontal plane and for swimming motion while in the simulation studies the crawling motion on a horizontal plane and on a slope is handled. An important disadvantage of this study is that the optimization method used has the potential to get trapped into local optima. In [2], genetic algorithm (GA) is used to find forward head serpentine (FHS) gait parameters maximizing the velocity of the robot by keeping the angular changes of the head link in an acceptable range. However, the energy consumption is not considered in this study. GA is also used in [3] to optimize CPG parameters and connection weights in terms of velocity of the snake robot and furthermore a fuzzy logic tuner is designed to maintain optimal locomotion in different environmental conditions in this study. In [4], optimal CPG parameters are discussed to achieve the efficient locomotion of the snake robot under different friction or slope. A criterion for locomotion efficiency is described as the ratio between forward displacement and energy consumption. In another study, parameters of a CPG based locomotion controller are obtained using GA according to changing ground friction for adaptive locomotion of snake robot [5]. Three different environments whose tangential friction coefficients are same is used to test lateral undulation locomotion in the study. In [6], three different gaits of the snake robot including lateral undulation, sidewinding locomotion, and sinus-lifting motion are analysed at eight different environments in terms of trade-off between locomotion speed of the robot and energy efficiency. For different friction coefficients, Pareto curves between locomotion speed and efficiency are obtained for all three gaits by 1500 random combinations of amplitude and frequency parameters in a given range of values. For underwater snake robots, effect of each parameter defining the motion pattern such as the amplitude, frequency and phase offset parameters on the consumed energy and the forward velocity of the robot are separately investigated [7]. In this study, cost of transportation (COT) index is used to define energy efficient motion. In [8], an optimization framework for solving a multi-objective optimization problem in order to obtain optimal gait parameters is proposed for underwater snake robot and particle swarm optimization (PSO) is used to investigate the energy efficiency of these robots. Optimal gait parameters are obtained for two different motion pattern, the lateral and eel-like motion of the underwater snake robot. An extension of the optimization framework proposed in [8] is presented in [9] for snake robots both on land and in water. In [10], to optimize locomotion efficiency, the locomotion parameters of a snake robot controlled by CPG are investigated considering the speed and energy consumption of the robot. The locomotion parameters are optimized with the cuckoo search (CS) algorithm for environments with different space widths and different ratios between friction coefficients in the tangential and normal directions. The aim of the optimization is to maximize the locomotion efficiency of the snake robot obtained by dividing the displacement of the robot by the energy consumption. Recently, a reinforcement learning (RL) based controller for generating locomotion gaits of the snake robot using the proximal policy optimization (PPO)
algorithm is proposed in [11]. Moreover, the grid search and Bayesian optimization algorithms are used to optimize the parameter set of the motion in this study. An expanded version of this study is presented in [12] by using the adversarial inverse reinforcement learning (AIRL) algorithm.

The literature review shows that the energy efficient locomotion problem of the snake robot is generally handled considering only maximization of the velocity of the robot. Although there are some studies considering both velocity of the robot and energy consumption, they have generally combined these two objectives into a single objective function by multiplied them with a weight factor. This approach called the weighted sum method has some disadvantages. The first of these is that selecting these weights precisely and accurately is very important and a difficult task since the weights effect the priority of the objectives in the objective function. The weights are generally determined uniformly distributed. However, this is not always guaranteeing a uniformly distributed of Pareto optimal solutions. The second disadvantage is that obtaining a Pareto front curve requires that the algorithm should be run multiple times with different weights. This is a time-consuming and an exhaustive process. Another disadvantage of this approach is that in mixed optimization problems including both minimization and maximization, the objectives should be converted to same type. Due to these disadvantages, it is more suitable that energy efficient locomotion problem is addressed as multi-objective optimization problem to obtain Pareto optimal solution set between the two objectives.

Evolutionary computation (EC) techniques for solving multi-objective problems are more useful since they are able to generate a set of multiple solutions in a single optimization run. For different kinds of optimization problems, there are a lot of different EC techniques [13-16]. Among these techniques, symbiotic organism search (SOS) algorithm which simulates the symbiotic interaction strategies between organisms in an ecosystem has increasingly become popularity in recent years because it presents more robust results with a faster convergence speed for optimization problems in various domains [17]. The superiority of the SOS algorithm over other EC algorithms is due to its three-stage strategy (mutualism, commensalism, and parasitism) used for updating the solutions. The first two stages of the SOS algorithm focus on exploration, and the two-stage exploration ensures the algorithm to center upon the region where the best solution is located. Moreover, the operation of random dimension mutation in the parasitism phase results in the better solution by helping jump out of local optima [18]. Another advantage of the SOS algorithm is that it is not necessary any specific algorithm parameters unlike most other metaheuristic algorithms. Therefore, the problem of trapping to local optima resulting from improper parameter setting is removed. This property of the algorithm also provides to reduce computational time [19].

This article is an expanded version of [20] in which the weighted sum method based multi-objective symbiotic organism search (MOSOS) algorithm was used to obtain optimal locomotion of the snake robot. In this paper, a fast non-dominated sorting multi-objective symbiotic organism search (FNSMOSOS) algorithm is proposed to find the parameters of the most efficient motion pattern for snake robot. FNSMOSOS is a very powerful algorithm at finding the optimal trade-off between objectives because it combines all the advantages of SOS algorithm with two important techniques such as fast non-dominated sorting technique generating the solution as close to the Pareto optimal solution as possible and crowding distance technique ensuring diversity in
solution. The optimal trade-off between forward velocity of the snake robot and average power consumption are obtained for lateral undulation motion which is the most common snake robot locomotion. From the results, it is seen that the proposed method has ability to produce promising results on obtaining the optimal forward velocity to achieve lower power consumption. When compared FNSMOSOS with the weighted sum method based MOSOS, it is seen that solutions obtained by FNSMOSOS are more uniformly distributed along the Pareto front. Moreover, FNSMOSOS provides a larger set of different solutions to decision maker only in a single run. Thus, the results can be used as a guide for control design of the snake robot.

For energy efficient locomotion, snake robots should adapt their motions to environments with different friction conditions, just as biological snakes do. Therefore, in this paper, the effectiveness of the proposed method in finding optimal gait parameters of snake robot is also investigated when the snake robot moves in different environmental condition. In order to test the reliability and robustness of FNSMOSOS in improving adaptive locomotion of the snake robot, environments representing a fairly wide friction range from glass to wood are used. The obtained results show that FNSMOSOS contributes the snake robot to maintain optimal locomotion in different environmental condition. When considering motion efficiency of snake robots is less than other mobile robots due to high friction, these results are very important for environmental adaptability of the snake robot.

The rest of this paper is organized as follows. In Section 2, the dynamic model of the snake robot is presented. The application of the proposed algorithms for energy efficient locomotion problem are explained in detail in Section 3 and the obtained results are reported and discussed in Section 4. Finally, the main conclusions are summarized in Section 5.

2. Snake Robot Modelling

In this section, equations of motion of the snake robot are briefly presented. For more details, see [21] and [22]. In Fig.1, snake robot diagram with n rigid links and its kinematic parameters are seen.

The link is of mass \( m_i \), length \( 2l_i \), and moment of inertia \( J_i = (m_i l_i^2/3) \). Each link is interconnected by \( n-1 \) motorized joints. Link angle \( \theta_i \) is defined as angle between link \( i \) and the global x axis while joint angle \( \phi_i \) is the difference between the link angles of two neighboring links and defined by \( \phi_i = \theta_i - \theta_{i+1} \). \((x_i, y_i)\) describe global coordinates of the center of mass (CM) of link \( i \), while \((p_x, p_y)\) are global coordinates of the CM of the robot. Fig. 2 shows forces and torques acting on the link \( i \) of the snake robot. These forces are ground friction force \((f_{R,i,x}, f_{R,i,y})\) and joint constraint forces \((h_{x,i}, h_{y,i})\) \((-h_{x,i-1}, h_{y,i-1})\) from link \( i+1 \) and link \( i-1 \), respectively. \( u_{i-1} \) and \( u_i \) are actuator torques exerted on link \( i \) from link \( i-1 \) and link \( i+1 \), respectively. The following given matrices and vectors are used in motion equations of the snake robot.
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\[ A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 \end{bmatrix}_{(n-1) \times n}, \quad D = \begin{bmatrix} 1 & -1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 \end{bmatrix}_{(n-1) \times n} \]

\[ E = \begin{bmatrix} e & 0_{n \times 1} \\ 0_{n \times 1} & e \end{bmatrix}, \quad e = [1, \ldots, 1]^T. \]

\[ \sin \theta = [\sin \theta_1, \ldots, \sin \theta_n]^T, \quad \cos \theta = [\cos \theta_1, \ldots, \cos \theta_n]^T, \quad \dot{\theta} = [\dot{\theta}_1, \ldots, \dot{\theta}_n]^T. \]

\[ S_\theta = \text{diag} (\sin \theta), \quad C_\theta = \text{diag} (\cos \theta), \quad M = \text{diag} (m_1, \ldots, m_n), \quad J = \text{diag} (J_1, \ldots, J_n). \]

**Fig. 1.** $n$ links snake robot diagram

**Fig. 2.** The forces and torques acting on each link of the snake robot

The position of the snake robot $p$ defined by its global CM is given in (1).
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\[
p = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} \frac{1}{m} \sum_{i=1}^{n} m_i x_i \\
\frac{1}{m} \sum_{i=1}^{n} m_i y_i \end{bmatrix} = \frac{1}{n} \begin{bmatrix} e^T X \\
e^T Y \end{bmatrix}.
\]  

(1)

where \( m \) is sum of mass of each link, \( X = [x_1, \ldots, x_n]^T \) and \( Y = [y_1, \ldots, y_n]^T \).

Frictions in normal and tangential directions between the snake robot and the ground play a crucial role especially during lateral undulation motion of the robot [23]. The friction forces on all links (\( f_R \)) are given in (2) according to viscous friction model. A distributed contact model is adopted in the formulation of friction force.

\[
f_R = \begin{bmatrix} f_{R,n} \\ f_{R,t} \end{bmatrix} = \begin{bmatrix} C_{\theta} - S_{\theta} & 0 \\
S_{\theta} & C_{\theta} \end{bmatrix} \begin{bmatrix} C_{\theta} M & 0 \\
0 & C_{\theta} M \end{bmatrix} \begin{bmatrix} C_{\theta} S_{\theta} \\
S_{\theta} \end{bmatrix} \begin{bmatrix} \dot{X} \\
\dot{Y} \end{bmatrix}.
\]

(2)

where \( c_t \) and \( c_n \) represent friction coefficients in tangential and normal directions of the link \( i \in \{1, \ldots, n\} \), respectively. \( X \) and \( Y \) are the linear velocities of the links.

The torque applied to CM of the link because of friction force can be defined as in (3).

\[
\tau_R = -C_n \dot{\theta}.
\]

(3)

The dynamic model of the snake robot is described in matrix form as in (4).

\[
\begin{bmatrix} M_{\theta} & 0 \\ 0 & mI \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{p} \end{bmatrix} + \begin{bmatrix} W \dot{\theta}^2 \\ 0 \end{bmatrix} - \begin{bmatrix} C \dot{\theta} \\ 0 \end{bmatrix} = \begin{bmatrix} D^T \\ 0 \end{bmatrix} u.
\]

(4)

3. Optimization of Motion

An optimization framework for the efficient motion of the snake robot is presented in this paper. The block diagram of the proposed method is given in Fig. 3. The parts of the system are the snake robot model in contact with the environment presented in Section II, a motion pattern generator, a joint controller, and an optimization algorithm. For different gait parameters, the optimization algorithm evaluates the objective functions calculated by using the forward velocity and average power consumption obtained by simulating the dynamic model of the snake robot.
3.1. Motion Pattern Generator

Lateral undulation is the most common seen gait in almost all snake species. To achieve this motion pattern for snake robot, a sinusoidal reference signal given in (5) is applied to each joint of the snake robot [24].

$$\phi_{i,ref} = \alpha \sin(\omega t + (i-1)\beta) + \gamma.$$  \hspace{1cm} (5)

where $\alpha$, $\omega$ and $\beta$ are parameters used to determine amplitude, angular frequency, and phase shift between the neighbourhood joints of the gait pattern. The joint offset used to control the direction of the locomotion is chosen as $\gamma = 0$ in this study.

3.2. Joint Controller

A PD controller is used to let the snake robot track the reference joint angle $\phi_{ref}$ and the control input for joint $i$ is given in (6). The derivative of $\phi_{ref}$ is obtained by passing the reference signal through a second order low pass filter [22]. The filter parameters are set to $\omega_n=25$ and $\xi=1$ in this study.

$$u_i = k_p(\phi_{ref} - \phi) + k_D(\dot{\phi}_{ref} - \dot{\phi}).$$  \hspace{1cm} (6)

where $k_p > 0$ and $k_D > 0$ are controller gains.

3.3. Optimization Algorithm

In this paper, two different multi-objective SOS based algorithms are presented for energy efficient locomotion of the snake robot. These algorithms are the weighted sum method based multi-objective SOS algorithm and a fast non-dominated sorting multi-objective SOS algorithm. Details of these algorithms are given below.
An overview of the symbiotic organism search (SOS) algorithm

Developed by inspiring from the symbiotic interaction strategies between organisms in the ecosystem, SOS algorithm is a robust and effective metaheuristic method [25]. Like other heuristic optimization techniques, SOS starts with a randomly produced population of organisms named as the ecosystem and each organism representing a candidate solution is iteratively used to find optimal global solution. The algorithm consists of three phases, namely mutualism, commensalism and parasitism which are biological interactions seen mostly in the real world. Three phases of SOS algorithm are carried out for each organism selected in order by beginning from the first organism $X_i$ in the ecosystem. The organism $X_i$ interacts with a different organism $X_j$ randomly selected from the ecosystem in all three phases. According to the type of the interaction, the new candidate solutions are generated in the three phases and in case these new solutions are better than previous ones, the old solutions are updated. The process is repeated until the stopping criterion, which is the maximum number of iterations, is achieved.

**Mutualism phase.** In this phase, because both organisms provide advantage from the association, the new candidate solutions for both $X_i$ and $X_j$ organisms are generated as formulated in (7) and (8). Both organisms do not benefit equally from the interaction. $BF_1$ and $BF_2$ parameters are used in the equations to reflect this situation and selected randomly 1 or 2.

\[
X_{\text{new}} = X_i + \text{rand}(0,1) \times (X_{\text{best}} - MV \times BF_1).
\]

\[
X_{\text{new}} = X_j + \text{rand}(0,1) \times (X_{\text{best}} - MV \times BF_2).
\]

where the $X_{\text{best}}$ is the organism which has best fitness value in the ecosystem. $MV$ is the mutual vector and defined as in (9).

\[
MV = \frac{X_i + X_j}{2}.
\]

**Commensalism phase.** In commensalism phase, only one organism $X_i$ benefits and the other $X_j$ is not affected. Hence, the new candidate solution only for $X_i$ is produced as in (10).

\[
X_{\text{new}} = X_i + \text{rand}(-1,1) \times (X_{\text{best}} - X_j).
\]
Parasitism phase. In parasitism phase, one of the two different species benefits from the other by damaging it. A parasite vector is created by duplicating $X_i$ and then modifying it randomly in the search space. $X_i$ is selected randomly from the ecosystem and used to serve as a host to the parasite vector.

The weighted sum method based multi-objective symbiotic organism search algorithm

This method considers a multi-objective optimization problem as a single-objective optimization problem. This single objective function is created by summing each objective function multiplied with a weight factor. The sum of the weights of all objectives should be 1.

The energy efficient locomotion problem of the snake robot is a mixed optimization problem including both maximization of the forward velocity of the snake robot and minimization of the average power consumption. For this reason, these objectives should be converted into one type while combining them into a single objective function. The single objective function used in this study is given in (11).

$$J = (1 - w) (P_{avg})_w - w (v_f)_w,$$  \hspace{1cm} (11)  

where $P_{avg}$ is the average power consumption and $v_f$ is the forward velocity of the snake robot. These indices are calculated as in (12) and (13), respectively. $w$ is the weight factor and determines priority of the $P_{avg}$ and $v_f$ in the objective function. The subscript $sc$ represents the scaled values of power consumption and forward velocity.

$$P_{avg} = \frac{1}{T} \int_0^T \left[ \sum_{i=1}^{n_d} u_i(t) \dot{\phi}_i(t) \right] dt,$$  \hspace{1cm} (12)  

where $T$ is the simulation time. The actuation torque $u_i$ are calculated given as in (6) while the angular velocity for joint $i$ is found by using derivative of the expression $\phi_i = \theta_i - \theta_{i+1}$.

$$v_f = \frac{\sqrt{(p_x(T) - p_x(0))^2 + (p_y(T) - p_y(0))^2}}{T},$$  \hspace{1cm} (13)  

where $(p_x(0), p_y(0))$ and $(p_x(T), p_y(T))$ denote initial and final positions of CM of the snake robot.

$P_{avg}$ and $v_f$ should be firstly scaled by dividing by their maximum values as defined in (14). The maximum values are determined by performing the optimization at $w=1$.

$$\left( P_{avg} \right)_{max} = \frac{P_{avg}}{\left( P_{avg} \right)_{max}}, \quad \left( v_f \right)_{max} = \frac{v_f}{\left( v_f \right)_{max}}.$$  \hspace{1cm} (14)  

The ecosystem matrix is obtained as in (15) by producing $n_o$ random organisms with dimension $n_d$ within the lower and upper bounds. Each element of the organism vector indicates parameters of the gait to be applied to the snake robot. In this problem, the dimension of the organisms is determined as 3 because the gait parameters to be
optimized are $\alpha$, $\beta$ and $\omega$. The fitness value of each organism in the ecosystem is calculated according to the objective function in (11).

$$
\text{ecosystem} = \begin{bmatrix}
\text{organism}_1 \\
\vdots \\
\text{organism}_n 
\end{bmatrix} = \begin{bmatrix}
\alpha_{1,1} & \beta_{1,2} & \omega_{1,3} \\
\vdots & \ddots & \vdots \\
\alpha_{n,1} & \beta_{n,2} & \omega_{n,3}
\end{bmatrix}.
$$

In conclusion, this optimization problem can be expressed as that the objective function in (11) is minimized subject to bound constraints given in (16) representing limitations of the servo motor and the parameters of the sinusoidal motion pattern. If these values go over the limits for any organisms, the corresponding organism is removed from the ecosystem by determining its fitness value as a high value.

$$
\min_{\alpha, \beta, \omega} J \equiv \left[ \frac{P_{\text{avg}}}{V_f} \right] \\
\text{s.t.: } \left| \phi_{t_{\text{ref}}} \right| \leq \phi_{\text{max}}, \left| \phi_{t_{\text{ref}}} \right| \leq \phi_{\text{max}}, |u_i| \leq u_{\text{max}} \\
0 \leq \alpha \leq \alpha_{\text{max}}, 0 \leq \beta \leq \beta_{\text{max}}, 0 \leq \omega \leq \omega_{\text{max}}
$$

**Fast non-dominated sorting multi-objective symbiotic organisms search algorithm (FNSMOSOS)**

FNSMOSOS presents a set of Pareto optimal solutions which is non-dominated with respect to each other instead of a unique optimal solution like in the weighted sum method based MOSOS. It uses fast non-dominated sorting (FNS) technique and crowding distance (CD) technique to preserve elitism and maintain diversity. The flowchart of proposed FNSMOSOS algorithm to solve the optimal locomotion problem of the snake robot is given in Fig. 4. This algorithm initializes with an ecosystem which is generated in the same way as the weighted sum method based MOSOS. For each organism, two separate objective functions defined in (12) and (13) used to minimize the average power consumption and maximize the forward velocity of the snake robot are evaluated and the fitness values of the any organisms which do not satisfy the constraints described in (16) are set to a high value. The new candidate organisms are generated in mutualism, commensalism, and parasitism phases. These phases are carried out as in the weighted sum method based MOSOS. The only difference is to be also move the dominated organisms to an advanced ecosystem for selecting next generation ecosystem alongside the non-dominated organisms are kept in the current ecosystem as a result of the comparing the fitness values of new organisms with their previous fitness values. Afterwards, the current ecosystem ($n_o$) and the advanced ecosystem ($4n_o$) are combined. The obtained combined ecosystem ($5n_o$) is sorted by using FNS technique to select the best $n_o$ organisms for next generation.
In FNS technique, the organisms are grouped into fronts by comparing fitness values of each organism with all other organisms in the ecosystem. For determining front of each organism, the first step is to calculate domination count $n_p$ defined as the number of organisms that dominate the organism $p$, and $S_p$ which is a set of organisms dominated by organism $p$. Each organism with $n_p = 0$ is assigned to first front ($F_1$) also known as Pareto front and they are better than others for one objective at least. The next step is to visit $S_p$ for each organism belonging to the Pareto front and reduce the domination count.
of each organism in $S_p$ by one. Thus, the organisms whose $n_p$ becomes zero are assigned to second front (F2). This operation goes on until all organisms in the ecosystem are assigned to a front. A general scheme of non-dominated sorting procedure is shown in Fig. 5. More details of the fast non-dominated sorting technique can be found in [26].

After FNS technique is applied, each front beginning from the first front is assigned to the new ecosystem one by one until the ecosystem size reaches up to $n_o$. If the addition of an entire front to the ecosystem causes the size of the ecosystem to exceed, the best organisms in this front are selected by crowding distance technique. For this, all organisms in this front are firstly sorted in ascending order for each objective function. Thereafter, the CD values of boundary organisms with smallest ($i=1$) and largest ($i=l$) fitness values are assigned to an infinite value and for other intermediate organisms ($i=2$ to $l-1$), the CD value is calculated as normalized difference in fitness value of two neighbouring organisms ($i+1$ and $i-1$), given as in (17). This computation is performed for each objective function $j$ ($j=1,2,\ldots,m$) and the total crowding distance value of the organism $i$ is found by summing the individual distance values corresponding to each objective function, given as in (18). For a problem involving two objective functions, computation of the crowding distance for the organism $i$ is shown in Fig. 6.

$$d_i^j = \frac{f_i^{j+1} - f_i^{j-1}}{f_{\text{max}}^j - f_{\text{min}}^j}.$$  \hspace{1cm} (17)

$$d_i = d_i^1 + d_i^2 + \ldots + d_i^m.$$ \hspace{1cm} (18)

![Fig. 5. A general scheme of non-dominated sorting procedure](image)
4. Optimization of Motion

In this section, the results of the proposed methods for energy-efficient locomotion of the snake robot are presented. A five link wheel-less snake robot which has identical links is used in this study and the parameters of the robot modelled are given in Table 1.

Table 1. Model parameters of the snake robot

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of links</td>
<td>$n = 5$</td>
</tr>
<tr>
<td>The length of a link</td>
<td>$2l = 0.18 \text{ m}$</td>
</tr>
<tr>
<td>Mass of each link</td>
<td>$m = 0.8 \text{ kg}$</td>
</tr>
<tr>
<td>Moment of inertia of each link</td>
<td>$J = 0.00216 \text{ kgm}^2$</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>$c_t = 0.1; c_n = 10$</td>
</tr>
<tr>
<td>The parameters of the PD controller</td>
<td>$k_p = 20; k_d = 5$</td>
</tr>
</tbody>
</table>

4.1. Performance of the weighted sum method based MOSOS

The first method used to obtain the optimal locomotion of the snake robot is the weighted sum method based MOSOS algorithm. The initial parameters of the algorithm are set as in Table 2. The dimension of the problem should be equal to the numbers of parameters to be optimized. Thus, this parameter is determined as 3 for energy efficient locomotion of snake robot. Because the determination of the number of organisms in the ecosystem is not based on any general rule, it is empirically set to $n_o = 15$. Similarly, the maximum iteration number is also empirically selected as $N = 20$ by considering its effect on the finding the optimal solution. The physical constraints of joints are determined based on the servo motor (HSR-5990TG) while the lower and upper bounds of the search space are determined according to the minimum and maximum values of the
parameters of the sinusoidal motion pattern. For the scaling of $P_{avg}$ and $v_f$ in the objective function, their maximum values are found as $v_f = 0.39 \text{ m/s}$ and $P_{avg} = 1.20 \text{ W}$, respectively.

**Table 2.** Optimization parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension of organism</td>
<td>$n_d = 3$</td>
</tr>
<tr>
<td>Number of organisms</td>
<td>$n_o = 15$</td>
</tr>
<tr>
<td>Iteration number</td>
<td>$N = 20$</td>
</tr>
<tr>
<td>Upper bounds of the gait parameters</td>
<td>$\alpha_{max} = 90^\circ, \beta_{max} = 90^\circ, \omega_{max} = 210^\circ/\text{s}$</td>
</tr>
<tr>
<td>The physical constraints of joints</td>
<td>$\phi_i^{\text{max}} = 90^\circ, \dot{\phi}_i^{\text{max}} = 429^\circ/\text{s}, u_i^{\text{max}} = 2.3 \text{ Nm}$</td>
</tr>
</tbody>
</table>

The forward velocity and the average power consumption are obtained by changing the weight factor between 0.1 and 1 with a step size of 0.1. The effect of weight factor on the forward velocity and the average power consumption are seen in Fig. 7 (a). From this figure, it is seen that the forward velocity increases as $w$ value increases and thus the average power consumption also increases due to the increasing velocity. The snake robot can achieve maximum forward velocity $v_f = 0.39 \text{ m/s}$ with the corresponding maximum average power consumption $P_{avg} = 1.20 \text{ W}$. As it was expected, the maximum values are obtained while $w = 1$. A set of Pareto optimal solutions obtained is seen in Fig. 7 (b). This figure indicates that the obtained solution set has a good coverage but not distributed very uniformly. This finding proves that the weights uniformly distributed cannot always present a uniform distribution of the Pareto optimal solutions.

The obtained results for the optimal gait parameters for each weight factor are also presented in Table 3. As seen in Table 3, the average power consumption of the snake robot significantly decreases from $P_{avg} = 1.20 \text{ W}$ to $P_{avg} = 0.42 \text{ W}$ when the weight factor is changed from $w = 1$ to $w = 0.6$. On the other hand, there is only a small decrease in the forward velocity from $v_f = 0.39 \text{ m/s}$ to $v_f = 0.34 \text{ m/s}$. Hence, a 65.08% reduction in the average power consumption of the snake robot can be obtained by sacrificing a 13.77% reduction in the forward speed. If a lower reduction in forward velocity is desired, by choosing the gait parameters at $w = 0.8$ in which the forward velocity decreases only by 4.11%, the average power consumption can be reduced by 43.27%. This table is useful for decision makers considering system requirements and can be used to find the optimal trade-off between the forward velocity of the snake robot and the average power consumption.
The change of the forward velocity and average power consumption depending on the change of the optimal gait parameters in (5) are illustrated in Fig. 8. This figure shows that an increase of the parameter $\alpha$ also results in an increase of the forward velocity and the average power consumption. On the other hand, it is seen that the parameter $\beta$ has the opposite effect of the parameter $\alpha$. This means that the forward velocity and the average power consumption decrease as parameter $\beta$ increases. Another important finding obtained from Fig. 8 is that the optimal value of the parameter $\alpha$ is in the range from 15º to 35º, while parameter $\beta$ is in the range from 40º to 65º. When we examine the effect of parameter $\omega$ on the forward velocity of the snake robot, it is seen that the parameter $\omega$ is at the maximum value 210º/s for most of the weight factors.

### Table 3. Obtained results for the optimal gait parameters for each weight factor

<table>
<thead>
<tr>
<th>$w$</th>
<th>$\alpha$ (deg)</th>
<th>$\omega$ (deg/s)</th>
<th>$\beta$ (deg)</th>
<th>$v_f$ (m/s)</th>
<th>$P_{avg}$ (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1000</td>
<td>15.2200</td>
<td>70.9344</td>
<td>90.0000</td>
<td>0.0531</td>
<td>0.0125</td>
</tr>
<tr>
<td>0.2000</td>
<td>18.1374</td>
<td>125.4008</td>
<td>78.1864</td>
<td>0.1375</td>
<td>0.0601</td>
</tr>
<tr>
<td>0.3000</td>
<td>19.2169</td>
<td>145.2789</td>
<td>61.1747</td>
<td>0.2102</td>
<td>0.1337</td>
</tr>
<tr>
<td>0.4000</td>
<td>20.2889</td>
<td>189.7583</td>
<td>62.4138</td>
<td>0.2730</td>
<td>0.2377</td>
</tr>
<tr>
<td>0.5000</td>
<td>21.8587</td>
<td>208.3734</td>
<td>59.8795</td>
<td>0.3166</td>
<td>0.3450</td>
</tr>
<tr>
<td>0.6000</td>
<td>23.2913</td>
<td>210.0000</td>
<td>56.6467</td>
<td>0.3375</td>
<td>0.4207</td>
</tr>
<tr>
<td>0.7000</td>
<td>25.9909</td>
<td>210.0000</td>
<td>53.3036</td>
<td>0.3605</td>
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<td>0.8000</td>
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<td>0.9000</td>
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<tr>
<td>1.0000</td>
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<td>210.0000</td>
<td>42.6450</td>
<td>0.3914</td>
<td>1.2049</td>
</tr>
</tbody>
</table>
According to these observations, we can determine the lower and upper bounds of the parameter $\alpha$ and $\beta$ in a narrower range and the size of the search space can be reduced from 3 to 2 by setting $w$ to the maximum value. Thus, the optimization process applied to find optimal gait parameters for snake robot can become more efficient and less costly computation. Finally, the position of the CM of snake robot for each of the weight factor is presented in Fig. 9.
4.2. Performance of the FNSMOSOS

For finding the parameters of the most efficient motion pattern for snake robot, the second proposed method in this paper is FNSMOSOS. In this algorithm, the number of organisms in the ecosystem and maximum iteration number are 100 and 20, respectively. A set of Pareto optimal solutions obtained with FNSMOSOS is seen in Fig. 10. This figure demonstrates FNSMOSOS achieves convergence to optimal Pareto front with a good diversity. When compared this Pareto front with that in Fig.7(b), it is seen that a more uniform distribution of solutions is achieved on the Pareto front with FNSMOSOS.

![Fig. 10. Pareto optimal solutions obtained by FNSMOSOS and weighted sum based MOSOS](image-url)

In this algorithm, the maximum forward velocity is obtained \( v_f = 0.39 \) m/s as in the weighted sum method based MOSOS algorithm. The maximum average power consumption corresponding to maximum forward velocity is \( P_{avg} = 1.20 \) W. The obtained results for the some of Pareto optimal gait parameters are given in Table 4. As seen from the table, FNSMOSOS presents more set of different solutions. Therefore, decision maker can balance between the forward velocity of the snake robot and the average power consumption by selecting the optimal gait parameters among more options according to systems requirements. For example, solution 22 in comparison with the solution 30 could be a good option by providing a 51.23% reduction in the average power consumption with only a small decrease of 6.56% in the forward speed. If the available power of the system is more limited, the average power consumption can be reduced by 74.60% by choosing the gait parameters in the solution 17. In this case, the forward velocity decreases only by 22.74%.

The effect on forward velocity and average power consumption of Pareto optimal gait parameters are presented in Fig 11. These curves are like those in Fig. 8 obtained with the weighted sum method based MOSOS algorithm. This finding indicates that the two different algorithms based MOSOS produce stable results for optimal locomotion of the snake robot.
Table 4. Obtained results for some of Pareto optimal gait parameters

<table>
<thead>
<tr>
<th>Solution</th>
<th>$\alpha$ (deg)</th>
<th>$\omega$ (deg/s)</th>
<th>$\beta$ (deg)</th>
<th>$v_f$ (m/s)</th>
<th>$P_{avg}$ (W)</th>
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</thead>
<tbody>
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<td>42.8074</td>
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<td>1.2040</td>
</tr>
</tbody>
</table>

Table 5. Different environments

<table>
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<tr>
<th>Environment</th>
<th>$c_t$</th>
<th>$c_s$</th>
</tr>
</thead>
<tbody>
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<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>0.01</td>
<td>1.0</td>
</tr>
<tr>
<td>4</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
<td>1.0</td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>8</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>9</td>
<td>0.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>
In this paper, the optimal gait parameters for the adaptive locomotion of the snake robot in different environment conditions are also investigated by using FNSMOSOS. For this purpose, nine different environments representing different surfaces in a quite wide range from glass to wood are used. The friction coefficients representing the environments are given in Table 5 and the obtained Pareto optimal solutions for each environment are presented in Fig. 12.

As seen from Fig. 12, FNSMOSOS presents good performance for each environment in achieving optimal solutions converging to the Pareto front. As it was expected, the snake robot has reached to a higher forward velocity in environments where \( c_n \) is high such as environment 3, environment 6 and environment 9. Moreover, it is seen that if \( c_n \) is constant, the snake robot can move faster in environments where the ratio between the two friction coefficients \( c_n/c_t \) is high such as environment 3.
The effect of different friction coefficients acting in the direction normal and tangential to snake robot on the forward velocity and average power consumption according to the optimal gait parameters are presented in Fig.13-Fig.18. According to these figures, for the nine environments, the optimal value of the parameter $\alpha$ varies in the range between $20^\circ$ and $50^\circ$, while parameter $\beta$ varies in the range between $55^\circ$ and $90^\circ$. Although the parameter $\omega$ varies over a wider range between $50^\circ$ and $210^\circ$, the snake robot has achieved its maximum forward velocity when the parameter $\omega$ is at the maximum value $210^\circ/s$ in all environments. Moreover, these figures show that snake robot should modify its gait parameters to maintain its efficient locomotion in different ground conditions. The forward velocity and average power consumption in environments where $c_n$ changes while $c_t$ is constant are presented in Fig. 13, Fig. 14, and Fig. 15 according to optimal $\alpha$, $\beta$ and $\omega$ parameters, respectively. According to Fig. 13 and Fig. 15, the optimal $\alpha$ and $\omega$ parameters increase as $c_n$ decreases because the motion in slippery environments where $c_n$ is low requires more friction force. Based on these results, an important finding is concluded that snake robot should move with greater amplitude and frequency to get more friction force in forward direction in such environments. On the other hand, Fig 14 shows that the optimal $\beta$ parameter decreases in the same environment conditions. Similarly, Fig. 16, Fig. 17 and Fig. 18 demonstrate the forward velocity and average power consumption in environments where $c_t$ changes while $c_n$ is constant according to optimal $\alpha$, $\beta$ and $\omega$ parameters, respectively. The decreasing of $c_t$ effects the optimal gait parameters in the opposite direction of the decreasing of $c_n$. As $c_t$ decreases, the optimal $\alpha$ and $\omega$ parameters decrease while the optimal $\beta$ parameter increases. However, it is seen that the changing of $c_t$ has less of an effect on the optimal gait parameters than the changing of $c_n$.

Fig. 13. The effect of different friction coefficients acting in the direction normal to snake robot on the forward velocity and average power consumption according to the optimal $\alpha$ parameter.
Fig. 14. The effect of different friction coefficients acting in the direction normal to snake robot on the forward velocity and average power consumption according to the optimal $\beta$ parameter.
Fig. 15. The effect of different friction coefficients acting in the direction normal to snake robot on the forward velocity and average power consumption according to the optimal $\omega$ parameter

Fig. 16. The effect of different friction coefficients acting in the direction tangential to robot on the forward velocity and average power consumption according to the optimal $\alpha$ parameter
Fig. 17. The effect of different friction coefficients acting in the direction tangential to robot on the forward velocity and average power consumption according to the optimal $\beta$ parameters.

Fig. 18. The effect of different friction coefficients acting in the direction tangential to robot on the forward velocity and average power consumption according to the optimal $\omega$ parameters.
5. **Conclusion**

This paper has investigated the optimal gait parameters giving appropriate forward velocity for the lower power consumption of the snake robot. The necessary trade-off between the forward velocity and power consumption for optimally efficient locomotion of the snake robot is obtained by using two different algorithms based MOSOS. From the obtained results, it is seen that these two algorithms produce stable results for optimal locomotion of the snake robot. However, FNSMOSOS provides a better distributed solution set when compared with the weighted sum method based MOSOS. Moreover, it generates more different solutions only in a single run. Thus, the operators can easily determine and select the optimal operational strategy from the Pareto front based on the control targets and the available power of the snake robot. In this paper, efficient locomotion of the snake robot is also investigated by considering different environments having a fairly wide friction range. The obtained results are very important to the snake robot maintaining its optimal locomotion in different environmental condition. Thus, this study is useful for developing environmental adaptability and efficient motion of the snake robot which has low motion efficiency due to its friction dependent motion.

**References**

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