Revisiting Snapshot Algorithms by Refinement-based Techniques

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Abstract. The snapshot problem addresses a collection of important algorithmic issues related to distributed computations, which are used for debugging or recovering distributed programs. Among existing solutions, Chandy and Lamport have proposed a simple distributed algorithm. In this paper, we explore the correct-by-construction process to formalize the snapshot algorithms in distributed systems. The formalization process is based on a modeling language Event B, which supports a refinement-based incremental development using RODIN platform. These refinement-based techniques help to derive correct distributed algorithms. Moreover, we demonstrate how other distributed algorithms can be revisited. A consequence is to provide a fully mechanized proof of the resulting distributed algorithms.

Keywords: Distributed algorithms, correctness-by-construction, refinement, snapshot, verification.

1. Introduction

The snapshot problem is a fundamental aspect of distributed computations and distributed applications, since it produces a global state of a distributed system at a particular instant. It is a photography of a global state made up of local states of each process and communication channels. Several solutions for the snapshot problem have been published, among them we consider the seminal algorithm of Chandy and Lamport [13, 28, 30]. The snapshot computation is motivated by several applications as, for instance, the verification of stable properties like deadlock, successful termination and debugging of the distributed program using safe configuration. Snapshot algorithms constitute a pertinent collection of case studies for evaluating strengths and weaknesses of formal techniques like model-checking [14, 15] and theorem prover [14, 25, 29]. The correct-by-construction paradigm [18] offers an alternative approach to prove distributed algorithms and to derive the correct distributed algorithms through the reconstruction of a target algorithm using stepwise refinement and validated methodological techniques [2, 6, 20, 21]. It appears that the refinement is a key concept for organizing the re-development of an existing distributed algorithm [2] to discover a new set of distributed algorithms [9] by reusing or replaying with the former development.
In this paper, we focus on the distributed snapshots for specific problems. The prime objective is to solve a problem using refinement techniques and to provide an evidence of correctness of given solutions, which are obtained through the correct-by-construction process. We are mainly interested by providing recipes for using the Event B framework and refinement for developing the distributed algorithms. Massingill and Chandy[19] introduce archetypes for facilitating parallel program design; more recently, Chandy et al [12] propose the refinement of formal archetypes to produce verified distributed software using the theorem prover PVS. The conceptual idea of the archetypes is very close to the design patterns in the software engineering domain. Refinement plays a central role in the integration of different archetypes and constitutes the semantical glue for ensuring the correctness of the resulting process. This approach is based on the use of PVS, which is employed to prove the properties of problems modelled using archetypes. Our recipes are conceptually close to the notion behind the archetypes and our aims are to use the Event B framework for developing correct-by-construction distributed algorithms, and enrich a collection of complex distributed algorithms (Project RIMEL: http://rimel.loria.fr). Another objective is to show the power of the correct-by-construction process and our recipes through the re-development and derivation of already existing and correct snapshot algorithms like the Chandy and Lamport algorithm [13], the algorithm of Lai and Yang [16] or the algorithm of Morgan [23]. Finally, the snapshot problem is already considered as a case study for illustrating the strength of rewriting logic [24] and we think that our development may help a reader to understand the behavioral theory of snapshot algorithms.

**Our Contribution.** This paper contributes to demonstrate semantical relationships existing between various snapshot algorithms (algorithms of Chandy and Lamport [13], Lai and Yang [16], Morgan [23]) with the refinement of models. We start with an abstract initial specification of the snapshot problem and we enrich this specification gradually by a progressive and incremental refinement. Several refinement steps allow to capture the complete and desired behaviour of snapshot algorithms. The refinement of models is the key element allowing preservation of properties between the levels of abstraction.

Moreover, we propose an architecture based on the correction-by-construction paradigm, for conceiving algorithms dedicated to observation of global states of distributed systems. This capture of a global state of a distributed system is introduced in the architecture by a model OBSERVATION. Our architecture is reusable and extendable since algorithms can be conceived or studied by refining either the OBSERVATION model or more concrete models (algorithms) provided in our architecture.

**Organization of the Paper.** The paper is organized as follows. Section 2 presents related works on the design of distributed snapshots. Section 3 defines the snapshot problem in distributed systems. Section 4 introduces notations of Event B and the formal activities of a global system. Section 5 presents refinement-based development of the snapshot algorithm, where we describe the OBSERVATION model for stating what we have to compute. Section 6 introduces the computation of a snapshot in the ASYNC-PROCESS and SYNC-PROCESS models, which simulate the OBSERVATION model. The global architecture of the refinement-based design is similar to the classical distributed algorithms [13, 16]. Section 6 also compares the formal modelling of the snapshot algorithms [13, 16, 23]. Section 7 concludes this paper along with the future work.
2. Related Works

Several literatures [3, 10, 28, 30] report works on the design of algorithms for the observation of distributed systems. Jaggi et al. [3] have proposed a snapshot algorithm (Distributed Snapshot Algorithm for MANETs) DSAM, which is derived from the algorithm of Chandy and Lamport, for dealing with snapshots in a mobile ad-hoc network. Chalopin et al. [10] have produced an algorithm combining the Chandy and Lamport and the SSP [27] algorithms for detecting the termination of the snapshot. Yang and Marsland [30] have elaborated debugging frameworks for distributed systems using various existing snapshot algorithms (Venkatesan, Lai and Yang, Li, Radhakrishnan and Venkatesh, Spezialetti and Kearns Algorithm, Morgan). The main motivation of these works is the need of observing global states of distributed systems, for identifying stable properties such as termination and deadlock [10], creating breakpoints for recovery, debugging systems [30], or taking into account non-fixed/mobile networks [3]. These works also present elements for proving the correctness of the designed algorithms. However, these elements are only partial proofs and do not assert completely the correctness and safe behaviour of the algorithms.

All these works present new algorithms, based on well-known classical ones (Chandy and Lamport, Lai and Yang, Morgan, etc), for the observation of distributed systems. However, the semantical link between the new and classical algorithms is not formally established. These works also focus on conceiving correct algorithms able to produce correct/consistent snapshots. Yet, as far as we know, the existing works are mainly based on simulation, which cannot guarantee the safe behavior. In this study, we use the formal techniques to specify the snapshot algorithms to make sure of the safe behavior, correctness and consistency using safety properties.

3. The Snapshot Problem

This section presents an abstract overview of the snapshot problem, which helps to understand our proposed solution. We consider a message passing model which formulates a distributed algorithm using a finite set of processes and channels. A direct channel connects each pair of nodes and a list of transformations is attached to each node, which performs either local actions or communications actions. The communication mechanism is supposed to be reliable, which guarantees that the channel does not lose any data packets.

For each node (process), a set of events (send, receive and internal events) is defined. A partial ordering called local causal order (denoted $<_p$ for a process ($p$)), induced by the local sequentiality of each process is defined. The following relationship $e_i <_p e_j$, between two events $e_i$ and $e_j$ of a process ($p$), indicates that $e_i$ occurs before $e_j$. A cut $C$ of a local set of events is a subset of events satisfying the relationship: $\forall p \in P, e, f \in C \cdot f \in L \land e <_p f \Rightarrow e \in C$. $P$ is a set of processes and $L$ is a set of pre-shot events (happening before the cut $C$).

Another ordering called causal order (denoted $<$) is defined as well. It is the smallest relation containing the local causal orders ($<_p$) and satisfying the send/receive ordering between processes. The relationship $e_m < e_n$, between two events $e_m$ and $e_n$ of a distributed system, means that $e_m$ occurs before $e_n$:
1. If $e_m$ and $e_n$ are local to a process $(p)$, then $e_m <_p e_n$.
2. If $e_m$ represents the sending of a message, then $e_n$ formulates the receiving of the message.
3. There exists another event $e_k$, such that $e_m < e_k$ and $e_k < e_n$.

A consistent cut $C$ of a set of events of a distributed algorithm is a subset of events, which satisfies the following relationship: $\forall e, f \in C \cdot f \in L \land e < f \Rightarrow e \in C$.

A snapshot $S$ is a global state of a distributed system, which is defined by a set of local states of nodes, and a set of channels states, produced by either internal actions or communication actions. The snapshot $S$ is meaningful and feasible, if there exists an execution producing the global state, and a set of messages is successfully passed through each channel $(p \rightarrow q)$ of the distributed system, where a set of messages is sent by the node $(p)$ and the sending messages are received by the node $(q)$.

The following theorem [28] relates the notions of cut and snapshot:

**Theorem 1.** A snapshot $S$ induced by a cut $C$ is meaningful if, and only if, $C$ is consistent if, and only if, $S$ is meaningful.

The aim of the snapshot algorithm is to compute a global state of the system from the local states or equivalently a consistent cut. We investigate steps for deriving three well-known snapshot algorithms [13, 16, 23] using proof-assisted stepwise development.

### 4. Stepwise Design of Distributed Algorithms

The *correct-by-construction* paradigm promotes the development of algorithms using a progressive and incremental approach. The key concept is the refinement which provides linking between *discrete* models by preserving safety properties. The Event B modeling language designed by Abrial [1, 8] borrows features from formal modeling languages like UNITY [11], TLA+ [17], action systems [4, 5]; those modeling languages share common aspects and especially the refinement concepts. The Event B is supported by an open environment RODIN integrating formal features for developing discrete logico-mathematical models. The Event B provides structures for expressing the reactive systems as a set of actions called events and maintaining a list of assertions called (inductive) invariants. These invariants formulate safety properties. We express our design for modeling the distributed algorithms in the Event B using correct-by-construction approach, which is also our primary objective of this work. We recall basic concepts of the Event B modeling language [1] and a formal development tool called RODIN [26].

#### 4.1. Modelling Actions Over States

The event-driven approach [1, 8] is based on the B notation. It extends the methodological scope of basic concepts in order to take into account the idea of formal models. A formal model is characterized by a (finite) list $x$ of *state variables* possibly modified by a (finite) list of *events*; an invariant $I(x)$ states properties that must always be satisfied by the variables $x$ and maintained by the activation of the events. Here, we briefly recall definitions and principles of formal models and explain how they can be managed by tools [26].
Modifications over state variables are stated by events. An event $e$ has two main parts: a guard $\text{grd}(e)$, which is a predicate built on the state variables, and an action, which is a generalized substitution. An event $e$ can take one of the three normal forms described in the table 1 and is associated with a before-after predicate $BA(e)(x, x')$, which describes the event as a logical predicate expressing the relationship between values of the state variables just before $(x)$ and just after $(x')$ the “execution” of the event (see Table 1).

<table>
<thead>
<tr>
<th>Event $e$</th>
<th>Before-after Predicate $BA(e)(x, x')$</th>
<th>Guard $\text{grd}(e)(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEGIN $x : [(P(x, x')]$ END</td>
<td>$P(x, x')$</td>
<td>$\text{true}$</td>
</tr>
<tr>
<td>WHEN $G(x)$ $\text{THEN}$ $x : [(Q(x, x')]$ END</td>
<td>$G(x) \land Q(x, x')$</td>
<td>$G(x)$</td>
</tr>
<tr>
<td>ANY $t$ $\text{WHERE}$ $G(t, x)$ $\text{THEN}$ $x : [(R(x, x', t)]$ END</td>
<td>$\exists t : (G(t, x) \land R(x, x', t))$</td>
<td>$\exists x : G(t, x)$</td>
</tr>
</tbody>
</table>

Table 1. Event B events and proof obligations

Proof obligations (INV 1 and INV 2) are produced by the tool RODIN [26] from events in order to state that an invariant condition $I(x)$ is preserved. Their general form follows immediately from the definition of the before-after predicate, $BA(e)(x, x')$, of each event $e$ (see Table 1). Note that it follows from the two guarded forms of the events and this obligation can be trivially discharged in case of false condition of the guard. When this is the case, the event is said to be disabled. The proof obligation FIS expresses the feasibility of the event $e$ with respect to the invariant $I$.

4.2. Describing the Network and its Activities

A network of processes is simply defined by a set of processes $P$, a set of channels between processes, namely $C$. We assume that $M$ is a set of messages that can transit along channels. Each process may have a local state and a set of local states is $P\text{States}$. The communication network is modelled by a structure called NETWORK. The network is supposed to be fixed (channels are not modified or created or deleted) and connected.
4.3. Describing the Current System

The snapshot algorithm captures a set of actions modifying a set of variables, through the observation of the current distributed system. Hence, our modeling process states that the existing system simulates a new set of modifications in the current state. A model \textsc{system} describes the general activities of the distributed system.

The defined variables for this model \textsc{system} are as follows:

- \( o \) associates each process to the timestamp of its last operation.
- \( l \) describes the local state of each process.
- \( h \) contains the traces of the activities (\textit{history}) for each process.
- \( ch\) presents a set of messages that circulates inside channels.
- \( store \) depicts a set of messages that is stored by each process.
- \( send \) models the sending messages that are sent by the processes.

The activities of the distributed system depicted in the model \textsc{system} are as follows:

\begin{itemize}
  \item \textit{Internal} operations modify states and variables local to nodes. These activities are modelled by the following events:
  \begin{itemize}
    \item \textbf{InternalLocal} demonstrates the modification of a local state of a process \((p)\). A new state \((ns)\) is chosen non-deterministically for the process \((p)\).
    \item \textbf{InternalMessage} models the modification of the local set of messages of a process \((p)\).
  \end{itemize}
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\end{itemize}

The process \((p)\) deletes a message \((m)\) from its local set of messages \((\text{store}(p))\).

\begin{itemize}
  \item \textbf{Sending} defines the sending of a message \((m)\) by a process \((p)\), through a channel \((c)\). The message \((m)\) has not yet been sent by the process \((p)\) and is not circulating inside the channel \((c)\). Therefore, the process \((p)\) is able to send \((m)\) through the channel \((c)\) connecting the process \((p)\) to another process.
\end{itemize}
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- **Receiving** presents the receiving of a message \((m)\) by a process \((q)\), via a channel \((c)\). The message \((m)\) is inside the channel \((c)\) leading to the process \((q)\): The message \((m)\) is removed from the channel \((c)\) and is stored by the process \((q)\).

After each operation, the time-stamp \((o(p))\) of a process \((p)\) is incremented, and a trace of activities (either internal/local or external) is added to history \((h(p))\) of the process \((p)\). A new step expresses the observation of the current system by another process which is defined by a refinement of the current model. In the next section, we define the refinement and apply it for the observation.

5. Incremental Proof-Based Development

5.1. Model Refinement

The refinement of a formal model allows us to enrich a model in an incremental way which is the foundation of the correct-by-construction [18] approach. Refinement provides a way to strengthen invariants and to add details to a model. It is also used to transform an abstract model in a more concrete version by modifying the state description. This is done by extending the list of state variables (possibly suppressing some of them), by refining each abstract event into a corresponding concrete version, and by adding new events. The abstract state variables, \(x\), and the concrete ones, \(y\), are linked together by means of a, so-called, gluing invariant \(J(x, y)\). A number of proof obligations ensure that (1) each abstract event is correctly refined by its corresponding concrete version, (2) each new event refines \(skip\), (3) no new event takes control for ever, and (4) relative deadlock-freeness is preserved. Details of the formulation of these proofs follows.

We suppose that an abstract model \(AM\) with variables \(x\) and invariant \(I(x)\) is refined by a concrete model \(CM\) with variables \(y\) and gluing invariant \(J(x, y)\). If \(BA(e)(x, x')\) and \(BA(f)(y, y')\) are respectively the abstract and concrete before-after predicates of the same event, respectively \(e\) and \(f\), we have to prove the following statement, corresponding to proof obligation (1):

\[
I(x) \land J(x, y) \land BA(f)(y, y') \Rightarrow \exists x'. (BA(e)(x, x') \land J(x', y')).
\]

Now, proof obligation (2) states that \(BA(f)(y, y')\) must refine \(skip\) \((x' = x)\), generating the following simple statement to prove (2):

\[
I(x) \land J(x, y) \land BA(f)(y, y') \Rightarrow J(x, y').
\]

For the third proof obligation, we must formalize the notion of the system advancing in its execution; a standard technique is to introduce a variant \(V(y)\) that is decreased by each new event (to guarantee that an abstract step may occur). This leads to the following simple statement to prove (3):

\[
I(x) \land J(x, y) \land BA(f)(y, y') \Rightarrow V(y') < V(y).
\]

\[\oplus: \text{add elements to a model}; \ominus: \text{remove elements from a model}\]
Finally, to prove that the concrete model does not introduce additional deadlocks, we give formalisms for reasoning about the event guards in the concrete and abstract models: $\text{grds}(AM)$ represents the disjunction of the guards of the events of the abstract model, and $\text{grds}(CM)$ represents the disjunction of the guards of the events of the concrete model. Relative deadlock freeness is now easily formalized as the following proof obligation (4):

$$I(x) \land J(x, y) \land \text{grds}(AM) \Rightarrow \text{grds}(CM).$$

When one refines a model, one can either refine an existing event by strengthening the guard and/or the before-after predicate (effectively reducing the degree of non-determinism), or add a new event in order to refine the skip event. The feasibility condition is crucial for avoiding possible states which have no successor; for instance, the division by zero. Furthermore, such refinement guarantees that a set of traces of the refined model contains (up to stuttering) traces of the resulting model. The basic foundations of the Event B modeling language along with several case studies are available in [1, 7]. The language of generalized substitutions is very rich and allows us to express any relation between states in a set-theoretical context. The expressive power of the language leads to require helps for writing relational specifications and this is why we should provide proof-based patterns for assisting the development of Event B models.

### 5.2. General Schema for Refinement

The correct-by-construction approach is based on the use of refinement and on introducing new features in the formal models. The methodology is simply described by the following diagram, which advocates different steps for producing a distributed algorithm using the correct-by-construction approach.

- The context $C$ states properties of graphs.
- The machine $M_0$ expresses the problem to solve by a set of events stating a relation between initial and final states, for instance, the computation of a correct snapshot.
- The refinement of $M_0$ into $M_1$ presents the inductive property allowing to express the computation of the snapshot by each node.
- The refinement of $M_1$ by $IM$ prepares the localisation phase and may require more than one refinement step.
- The next refinement of $IM$ is a refinement for producing a set of events corresponding to the localisation of information.
- $DA$ is derived from the $M_2$; $mapping$ checks that $M_2$ can be translated into a distributed programming language.

However, we consider a more general schema for developing the snapshot problem, since the snapshot problem is solved by an algorithm which is able to compute the current distributed state. Next subsection starts the refinement process by introducing the first refinement related to the observation of the snapshot.
5.3. Introducing the OBSERVATION model

The OBSERVATION model refines the SYSTEM model and introduces the functionality, which is required by the snapshot problem: to compute a snapshot. It does not explain how to compute it but what it should compute.

A set of new variables is introduced to model the required behaviour as follows:

- Two variables $s$ and $r$ are defined for ordering the sending and receiving of messages:
  - $s$ associates sent messages with channels and timestamps. The variable helps the users to determine the channel in which a message is sent and the time of the sending operation.
  - $r$ associates received messages with channels and timestamps. This variable indicates the channel from which a message is received by a process and the time of the receiving operation.
- $cut$ contains the result of the snapshot.

The refined versions of the events Sending and Receiving are modified to take the variables $r$ and $s$ into account:

- Sending uses the variable $s$ to indicate the channel ($c$) in which a message ($m$) is sent and the sending time.
- Receiving records a message ($m$) and receiving time from the channel ($c$) into the variable $r$.

A new event Snapshot models an abstraction of the snapshot procedure and states that a consistent cut (obtained in one-shot), namely $acut$, is assigned to $cut$: a moving message is not allowed to be part of the snapshot, if origin of the message is outside of the $cut$ and its destination is inside of the $cut$. The event expresses the intention to specify the required solution. Further refinements are necessary for introducing the inductive process leading to a consistent cut. Others events are related to the previous models, which are

\[
\begin{align*}
\text{EVENT SnapShot} & \quad \text{ANY} \\
& \quad \text{act} \quad \text{WHERE} \\
& \quad \text{grod1:} \quad acut \in P \rightarrow N \\
& \quad \text{grod2:} \quad \forall p, q, i, j, m. \left( p \in P \land q \in P \land m \in M \land p \neq q \land \exists h(p) \land \exists h(q) \land i \in h(p) \land j \in h(q) \land i \leq acut(q) \land (i \rightarrow q) \rightarrow m \in s \land (p \rightarrow q) \rightarrow m \in r \right) \Rightarrow s \leq acut(p) \\
& \quad \text{grod3:} \quad \forall p \in P \Rightarrow acut(p) \in dom(h(p)) \\
& \quad \text{THEN} \\
& \quad \text{act1:} \quad cut := acut \\
& \quad \text{END} \\
\end{align*}
\]
indicated by dots. Due to space limitations, we have given a sketch of the modeling. A detailed formal development is available.

6. Architecture of the Design

Figure 1 presents the complete formal development, which starts from SYSTEM and NETWORK and progressively leads to the OBSERVATION. The models ASYNC-PROCESS and SYNC-PROCESS are derived from the model OBSERVATION and present two different ways of computing a consistent snapshot: ASYNC-PROCESS describes an asynchronous computation of a snapshot [13, 16], whereas SYNC-PROCESS depicts a synchronous way of constructing a snapshot [23].

Fig. 1. General Architecture of the Design

It should be noted that SYNC-PROCESS and ASYNC-PROCESS model respectively the algorithms of Morgan [23] and Lai and Yang [16]. The model LAI-PROCESS presents a more concrete version of the snapshot algorithm of Lai and Yang [16]. A context FIFO-NETWORK extends NETWORK and adds elements of FIFO queues and communications ordering to the network. The model of the algorithm of Chandy and Lamport [13] (FIFO-PROCESS) is refined from the model ASYNC-PROCESS and uses the context FIFO-NETWORK. The model LOC-FIFO-PROCESS is a refinement of FIFO-PROCESS and defines a more concrete and local version of the snapshot algorithm of Chandy and Lamport [13].

6.1. Computing a Synchronous Snapshot

The SYNC-PROCESS model (see Fig.2) is a refinement of OBSERVATION, and defines the synchronous construction of a correct snapshot \((pcut)\) gradually: an external parameter (e.g. a global clock) triggers the snapshot procedure for all the processes, at the same time. More precisely, the SYNC-PROCESS model describes the steps of the snapshot algorithm of Morgan [23], which is based on the availability of a global time for all the processes. New variables are introduced:

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4 http://www.loria.fr/~andriami/snapshot-comsis-pdf/project.html
Fig. 2. The SYNC-PROCESS Machine

- \textit{mark}_m\text{ contains a set of messages that is sent after the snapshot.}
- \textit{tm}\ represents the global time.
- \textit{cstate} records, for each process, the incoming \textit{pre-snapshot} messages.
- \textit{pstate}\ contains, for each process, the state of the process during the snapshot.
- \textit{pcut} is an intermediate variable presenting the construction of the snapshot.

The invariant (A) (see Fig. 2) defines constraints on these variables and states the consistency of the snapshot: If a message \( m \) is sent by a process \( p \) at a time \( i \), and received by a process \( q \) at a time \( j \) before the snapshot, then the time \( i \) belongs to the past of the cut.

Events describe the computation of the snapshot:

- \textit{Tick} models the flow of time.

\begin{verbatim}
EVENT Tick
WHEN
  grd1 : tm = FALSE
THEN
  act1 : tm := TRUE
\end{verbatim}

We model the fact that a predefined global time \( t \) (for triggering the snapshot) has been reached, by setting the value of \( tm \) to \textit{TRUE}.
ProcessingSnapshot demonstrates the simultaneous local snapshots of all the processes. When the global time \((t)\) (for triggering the snapshot) has been reached, all the processes record their local states and begin to save incoming pre-snapshot messages.

Snapshot describes the global snapshot. An abstract parameter \(acut\) (modelling the global state of the system) of the event is removed and replaced by concrete variable \(pcut\). This new variable \(pcut\) represents the global state of the system and its value is computed in previous event ProcessingSnapshot. When all the processes have recorded their local states, as well as all the incoming pre-snapshot messages, the global state \((pcut)\) of the system is saved into the variable \(cut\).

The events Sending and Receiving are refined and their refinements describe the pre-snapshot and post-snapshot activities of the system:

- **SendingBeforeCut**: This event models the sending of messages before the global time \((t)\) for processing the snapshot is reached. The actions of this event are similar to the actions of the abstract event Sending.
- **SendingAfterCut**: This event demonstrates the sending of messages after the local snapshot of a process \((p)\). The message \((m)\) is marked as being sent by the process \((p)\) after the local cut.
- **ReceivingPreCutMessages**: This event presents the receiving of pre-snapshot messages by a process \((q)\), after a local cut.
- **ReceivingPostCutMessages**: This event expresses the receiving of marked post-snapshot messages by a process \((q)\), after a local cut. The message \((m)\) received by the process \((q)\) is recorded as a pre-snapshot message.
- **ReceivingBeforeCut**: This event models the receiving of messages before the global time \((t)\) for processing the snapshot is reached. The actions of this event are the same as the actions of the abstract event Receiving.

The following sections describe the derivation of other algorithms for computing snapshots, in a complete local manner, without any global mean \([13, 16]\).
6.2. Computing an Asynchronous Snapshot

The ASYNC-PROCESS model (see Fig. 3) refines the OBSERVATION model, and presents the asynchronous construction of a correct snapshot \((pcut)\) step-by-step. A control message \((marker)\) is introduced along with events to separate pre and post-snapshot messages for describing the development steps of the snapshot algorithm:

**MACHINE** ASYNC-PROCESS

**INVARIANTS**

- \(A : \forall p, q, i, j, m \cdot (p \in P \land q \in P \land m \in M \land p \neq q \land p \in \text{dom}(pcut)) \land q \in \text{dom}(pcut) \land j \leq \text{peut}(q) \land i \leq \text{peut}(p) \land (p \rightarrow q \rightarrow j) \rightarrow m \in r \) \(\Rightarrow i \leq \text{peut}(p)\)
- \(B : \forall p, q, i, j, m \cdot (p \in P \land q \in P \land m \in M \land p \neq q \land p \in \text{dom}(pcut)) \land q \notin \text{dom}(pcut) \land i \leq \text{peut}(p) \land (p \rightarrow q \rightarrow j) \rightarrow m \in r \) \(\Rightarrow i \leq \text{peut}(p)\)
- \(C : \forall p, q, i, m \cdot (p \in P \land q \in P \land m \in M \land p \neq q \land p \in \text{dom}(pcut)) \land q \notin \text{dom}(pcut) \land (p \rightarrow q \rightarrow i) \rightarrow m \in s \land (p \rightarrow q \rightarrow j) \rightarrow m \in r \) \(\Rightarrow i \leq \text{peut}(p)\)

**EVENTS**

- EVENT StartingSnapshot...
- EVENT ProgressingSnapshot...
- EVENT Snapshot...
- EVENT SendingBeforeCut...
- EVENT SendingAfterCut...
- EVENT ReceivingPreCutMessages...
- EVENT ReceivingPostCutMessages...
- EVENT ReceivingBeforeCut...

**Fig. 3. The ASYNC-PROCESS Machine**

**EVENT StartingSnapshot**

ANY
- \(\text{node, ncutate} \)
WHERE
- \(g_{rd1} : \text{node} \subseteq C\)
- \(g_{rd2} : \text{ncutate} \subseteq C \rightarrow P(M)\)
- \(g_{rd3} : \text{initiator} \notin \text{dom}(pcut)\)
- \(g_{rd4} : \text{node} = \text{send\_mark} \)

THEN
- \(a_{cl1} : \text{send\_mark} := \text{node}\)
- \(a_{cl2} : \text{ncutate} := \text{ncutate} \)
- \(a_{cl3} : \text{pcut}(\text{initiator}) := \text{initiator}\)
- \(a_{cl4} : \text{putate}(\text{initiator}) := \text{true}(\text{initiator})\)

**EVENT ProgressingSnapshot**

ANY
- \(i, c, \text{node, ncutate} \)
WHERE
- \(g_{rd1} : i \in P \land c \in C\)
- \(g_{rd2} : \text{node} \subseteq C\)
- \(g_{rd3} : \text{ncutate} \subseteq C \rightarrow P(M)\)
- \(g_{rd4} : i \in \text{proj}(c) \land i \notin \text{dom}(pcut)\)
- \(g_{rd5} : c \in \text{send\_mark} \land \text{mark}(c) \subseteq \text{mark\_m}\)
- \(g_{rd6} : \text{node} = \text{send\_mark} \land \text{proj}(c) = \text{mark\_m}\)

THEN
- \(a_{cl1} : \text{pcut}(i) := \text{false}\)
- \(a_{cl2} : \text{putate}(i) := \text{true}(i)\)
- \(a_{cl3} : \text{ncutate} := \text{ncutate} \)
- \(a_{cl4} : \text{send\_mark} := \text{node}\)

- **StartingSnapshot**: A node \((\text{initiator})\) starts to build of the snapshot. The node \((\text{initiator})\) saves its local state. It begins to record the incoming messages \((\text{marker})\) and finally, this node \((\text{special})\) sends a message \((\text{marker})\) to all of its neighbouring nodes.

- **ProgressingSnapshot**: A node \((i)\) receives a message \((\text{marker})\) from the neighbouring node and it begins to record all the incoming messages. If the node \((i)\) receives all the messages before sending the message \((\text{marker})\), it records the local state and transmits the message \((\text{marker})\) to its neighbours.

- **Snapshot**: All the nodes have received a message \((\text{marker})\). For all the nodes, the messages sent to them before a message \((\text{marker})\), have been received. Finally, the global state of the distributed system is saved.
The model also introduces a set of properties for describing the consistency of the cut:

(A) If a message $m$ is sent by a process $(p)$ at a time $(i)$, and received by a process $(q)$ at a time $(j)$ before the snapshot, then the time $(i)$ belongs to the past of the cut.

(B) If a message $m$ is sent by a process $(p)$ (which has already performed a local cut) at the time $(i)$, received by a process $(q)$ (which has not yet performed a local cut) at a time $(j)$, then the time $(i)$ belongs to the past of the cut.

(C) If a message $m$ has been sent by a process $(p)$ to process $(q)$ at a time $(i)$ (before the receiving of a message (marker) by the process $(p)$), then the time $(i)$ belongs to the past of the cut.

The events Sending and Receiving are refined to distinguish pre-snapshot messages and/or activities from their post-snapshot counterparts:

- **SendingBeforeCut**: This event describes the sending of a message $(m)$ by a process $(p)$, before the local cut of the process $(p)$. The actions of this event are similar to the actions of the “normal” event Sending.

- **SendingAfterCut**: This event presents the sending of the message $(m)$ by the process $(p)$, which follows the local cut of the process $(p)$. The message $(m)$ is marked as being sent after the local cut of the process $(p)$.

- **ReceivingPreCutMessages**: This event demonstrates the receiving of the message $(m)$ (sent by the process $(p)$ before the local cut of the process $(p)$) by a process $(q)$, after receiving a message (marker) by the process $(q)$.

The incoming message $(m)$ is recorded by the process $(q)$.
Revisiting Snapshot Algorithms by Refinement-based Techniques

– ReceivingPostCutMessages: This event shows the receiving of a message \(m\) (sent by a process \(p\) after the local cut of the process \(p\)) by a process \(q\), after the process \(q\) has performed a local cut. The message \(m\) received by the process \(q\) is removed from the set \(\text{mark}_m\) of marked post-snapshot messages.

– ReceivingBeforeCut: This event describes the receiving of a message \(m\) (sent by a process \(p\) before the local cut of the process \(p\)) by a process \(q\), before the process \(q\) performs a local cut. The actions of this event are the same as the actions of the “normal” event Receiving.

6.3. Deriving Asynchronous Snapshot Algorithms

The Lai and Yang Algorithm The Lai and Yang algorithm [16] is a two-phases protocol: either (A) one special process (called initiator) initiates the snapshot, or (B) another process among non-initiator processes extends the snapshot. Due to their similarities, we will focus on phase (A), depicted by the following steps:

process (initiator):
step 1: record local state;
step 2: snapshot := 1;
step 3: begin to record incoming pre-snapshot messages;
step 4: to send a message : \(<\text{message}, \text{snapshot}>\);

Details of the two possible phases are described by the model ASYNC-PROCESS, an abstract model of the Lai and Yang algorithm [16]: channels between processes are represented by sets of messages; however a message \(m\) is extended by a bit, which determines either if the message is pre or post-snapshot. The bit is 1, when the predicate \(m \in \text{mark}(c)\) holds. The model LAI-PROCESS, refining ASYNC-PROCESS, localizes informations and describes a model for the Lai and Yang algorithm. We can identify, in the LAI-PROCESS model, events representing the phases (A) and (B) of the Lai and Yang algorithm:

The actions of this event can be associated with the steps of phase (A):

– act1 models step 1: the process (initiator) records its local state.
– act2 represents step 2: the process (initiator) takes a local snapshot.

– act3 indicates that the process (initiator) will record all pre-snapshot incoming messages (step 3).
– Finally, act4 and act5 match step 4: the process (initiator) indicates that all outgoing messages will be labelled with the bit 1.
We can see that the two phases (A) and (B) are modelled, respectively, by the events `StartingSnapshot` and `ProgressingSnapshot`. The other events do not describe parts of the Lai and Yang algorithm; they depict activities of the processes and the network (communications, computations, etc.).

The Chandy and Lamport Algorithm The Chandy and Lamport algorithm [13] uses a mechanism of coloring and propagation of a red color from a white one. A white message occurs before a snapshot and a red message occurs after the snapshot. We split the two kinds of messages using a variable `mark`, indicating, whether or not messages (`marker`) have been sent by processes. The abstract model `FIFO-PHCESS` of this algorithm refines the model `PROCESS`: it is an abstract model of the Lai and Yang algorithm. Behaviours of the model `FIFO-PHCESS` correspond to behaviours of the model `ASYNC-PHCESS`, thanks to refinement. However, the machine `FIFO-PHCESS` and context `FIFO-NETWORK` (extension of the context `NETWORK`) introduce new features: the separation between the pre and post-snapshot messages is implemented by a FIFO communication mechanism. Channels between nodes are transformed from sets of messages to FIFO queues. Because of the clear distinction between the pre and post-snapshot phases, the bit of membership defined in the Lai and Yang algorithm can be removed; which means that the messages are less complex. However, we can observe that a strong constraint is added: in the Chandy and Lamport algorithm, FIFO communication channels are mandatory. The `LOC-FIFO-PHCESS` model refines the `FIFO-PHCESS` model: the `LOC-FIFO-PHCESS` model localizes events and is producing the algorithmic form of the Chandy and Lamport algorithm.

6.4. Comparing the Formal Modelling of Snapshot Algorithms

We have studied two categories of snapshot algorithms: synchronous snapshot algorithms and asynchronous snapshot algorithms.

In this section, we compare these two types of snapshot algorithms according to two criteria: first, we make comparisons of formal development complexity; then, we compare their local and global characteristics.

<table>
<thead>
<tr>
<th>Model</th>
<th>Total</th>
<th>Auto</th>
<th>Interactive</th>
</tr>
</thead>
<tbody>
<tr>
<td>NETWORK</td>
<td>6</td>
<td>6</td>
<td>100%</td>
</tr>
<tr>
<td>FIFO-NETWORK</td>
<td>5</td>
<td>4</td>
<td>80%</td>
</tr>
<tr>
<td>SYSTEM</td>
<td>55</td>
<td>50</td>
<td>90.9%</td>
</tr>
<tr>
<td>OBSERVATION</td>
<td>41</td>
<td>37</td>
<td>90.24%</td>
</tr>
<tr>
<td>SYNC-PROCESS</td>
<td>45</td>
<td>41</td>
<td>91.11%</td>
</tr>
<tr>
<td>ASYNC-PROCESS</td>
<td>96</td>
<td>66</td>
<td>68.75%</td>
</tr>
<tr>
<td>LAI-PROCESS</td>
<td>85</td>
<td>46</td>
<td>54.12%</td>
</tr>
<tr>
<td>FIFO-PHCESS</td>
<td>229</td>
<td>12</td>
<td>5.24%</td>
</tr>
<tr>
<td>LOC-FIFO-PHCESS</td>
<td>5</td>
<td>4</td>
<td>80%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>567</td>
<td>266</td>
<td>46.91%</td>
</tr>
</tbody>
</table>

Table 2. Summary of Proof Obligations

Asynchronous snapshot algorithms seem at first glance more complex than synchronous ones: we can see in the previous sections that asynchronous algorithms possess more steps than synchronous algorithms. Asynchronous snapshot algorithms have two phases: 1) a snapshot initialisation phase (event `StartingSnapshot`) and 2) a progression phase (event `ProgressingSnapshot`); whereas, synchronous algorithms only have one phase: the simultaneous computation of the snapshot by the processes (event `ProcessingSnapshot`). Clearly, this difference affects the size of the models: obviously,
formal models of asynchronous snapshot algorithms are more complex, since they contain more events. The difference in complexity is underlined by the number of invariants needed to demonstrate the consistency of a snapshot: the ASYNC-PROCESS model requires a set of invariants (A, B, C, see Fig.3), while the SYNC-PROCESS model only necessitates one (A, see Fig.2). Furthermore, the table 2, presenting the proof obligations discharged either manually or automatically, confirms that modelling asynchronous algorithms is more complex: the total number of proofs (96) for these algorithms is more than twice as much as the number of proofs (45) for synchronous algorithms, and it should be noted that the number of manual proofs for asynchronous algorithms is far greater (30 vs 4). The following table sums up the complexity differences between synchronous and asynchronous snapshot algorithms.

<table>
<thead>
<tr>
<th>Complexity</th>
<th>Synchronous</th>
<th>Asynchronous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invariants (consistency of snapshot)</td>
<td>less (1)</td>
<td>more (3)</td>
</tr>
<tr>
<td>Number of Algorithmic Steps</td>
<td>less (1)</td>
<td>more (2)</td>
</tr>
<tr>
<td>Number of Events related To The Computation</td>
<td>less (1)</td>
<td>more (2)</td>
</tr>
<tr>
<td>Number of Proofs</td>
<td>less (45)</td>
<td>more (96)</td>
</tr>
</tbody>
</table>

Table 3. Complexity of the Models of Snapshot Algorithms

If we analyse their local and global characteristics, we can see that synchronous snapshot algorithms are generally based on the observation of external global elements, such as global time, while, on the other hand asynchronous snapshot algorithms only relies on the local resources and informations of the processes. The table 4 summarises these characteristics:

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Synchronous</th>
<th>Asynchronous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Local</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 4. Local and Global Characteristics of the Models of Snapshot Algorithms

In a nutshell, we say that the availability of global shared elements (e.g. global time) greatly simplifies the formal design and modelling of distributed snapshot algorithms [30]. However this simplification of design is gained at the expense of other qualities, like localisation.

7. Discussion, Conclusion and Future Work

The snapshot algorithm identifies global states in a distributed system. The result of our works on the snapshot problem is the discovery of a generic architecture which allows the derivation of various algorithms. The model SYSTEM provides an abstract view of a distributed system and the activities of its processes (computations, communications, etc.). This model is generic: computations, activities, etc. can be made more specific, according to the peculiarities of studied systems and can be refined following the same methodology preserving correctness. The model SYSTEM is refined by a model OBSERVATION, which introduces the notion of snapshot: an event models the global snapshot of the distributed
system. The development of the snapshot is organised from the models called \textsc{async}-\textsc{process} and \textsc{sync}-\textsc{process}, which express the underlying computation procedure and can be refined into several other algorithms. The key idea is to separate the pre-shots and the post-shots and the solution depends on assumptions on communications, namely channels and messages; the \textit{mark} variable is either a marker for a bit, a marker for \texttt{fifo} channels or a marker for global temporal aspects. The complexity of the development is measured by the number of proof obligations which are automatically/manually discharged (see table 2). The main difficulty of the development was the expression of a consistent snapshot in the machines \textsc{async}-\textsc{process} and \textsc{sync}-\textsc{process}, therefore the establishment of the refinement relation between these machines and the machine \textsc{observation}. A set of invariants (A,B,C) of the machine \textsc{async}-\textsc{process} (Fig.3) and the invariant (A) of the machine \textsc{async}-\textsc{process} (Fig.2) were the keys of the development, where the generated proof obligations were quite difficult to discharge. We also notice that the number of manual proof obligations increases dramatically for the model \texttt{fifo}-\textsc{process} (see table 3): this augmentation is due to the transformation of sets of messages into \texttt{fifo} queues of messages. In fact, we had to prove a lot of properties defining the proper behaviours of \texttt{fifo} queues and communications. Moreover, the snapshot algorithm is supposed to work while another process \texttt{system} is working; \texttt{system} is a model for another distributed system and the snapshot algorithm is an implementation of the observation of the current system. Contrary to the verification by theorem provers [24], our work provides an architecture for developing the snapshot algorithm using essential safety properties together with a formal proof that asserts its correctness.

In this paper, we have experimented on fixed networks. As a part of our future efforts we consider the global family of snapshot algorithms to give a very precise description of different solutions and to link between these algorithms, as we notice that the algorithm of Chandy and Lamport is obtained from the algorithm of Lai and Yang by adding a \texttt{fifo} communication. Moreover, we plan to integrate the snapshot algorithm with complex distributed systems like mobile networks.

References

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