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An Asymmetric Index to Compare Trapezoidal Fuzzy Numbers

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Abstract. In this paper, we present a tool to help reduce the uncertainty presented in the resource selection problem when information is subjective in nature. The candidates and the "ideal" resource required by evaluators are modeled by fuzzy subsets whose elements are trapezoidal fuzzy numbers (TrFN). By modeling with TrFN the subjective variables used to determine the best among a set of resources, one should take into account in the decision-making process not only their expected value, but also the uncertainty that they express. A mean quadratic distance (MQD) function is defined to measure the separation between two TrFN. It allows us to consider the case when a TrFN is wholly or partially contained in another. Then, for each candidate a weighted mean asymmetric index (WMAI) evaluates the mean distance between the TrFNs for each of the variables and the corresponding TrFNs of the "ideal" candidate, allowing the decision-maker to choose among the candidates. We apply this index to the case of the selection of the product that is best suited for a "pilot test" to be carried out in some market segment.

Keywords: Fuzzy sets, distance, resource selection, pilot test, subjective information, marketing.

1. Introduction

Currently, companies that commercialize a broad range of products, sometimes with high product rotation, tend to have departments specializing in consistently providing technically viable and economically feasible ideas.

The "bank of ideas" that such organizations have produces a multitude of reference points for the delivery of new goods or services. Virtually, any previously screened idea may be of commercial interest, but many of the failures in its commercialization may arise from presenting it to the wrong kind of client. The design of a product must meet customer requirements, making it essential to carry out a post-design verification test with a sample of the intended market segment, before the product's full-scale commercialization [18].

We have to remember that R&D departments typically have a set of different products, from which they must choose the one best suited to the targeted segment of the market.

The suitability of these products is evaluated with subjective variables, which makes the application of techniques based on fuzzy subsets a straightforward matter.

By means of the theory of fuzzy subsets [22], we can select the best among a group of candidates when information is subjective in nature or comes from expert processed statistical data. As an example of this line of research, we can observe the work of Chen and Wang [6] and its application to search for the perfect home [7], the application to databases developed by Yang et al. [21], the process carried out by the International Olympic Committee for the selection of the venue of the 1st Summer Youth Olympic Games [11], and the evaluation of traffic police centers performance by Sadi-Nezhad and Damghani [17]. This process is based on the comparison between fuzzy numbers, a line of research that have been a cornerstone of the fuzzy sets theory, and of which we can cite the work, for example, of Tran and Duckstein [19], of Zeng and Guo [23], of Zhang, Zhang and Mei [24], of Lee, Pedrycz and Sohn [14], and of Guha and Chakraborty [10].

In this same line of research, we offer a mean quadratic distance (MQD) function between trapezoidal fuzzy numbers (TrFN) that allows us to consider the case when a TrFN is wholly or partially contained in another. In traditional distance functions between TrFN, when a fuzzy number is totally contained in another there is a distance between them, and this distance is symmetric. Nevertheless, in our index, the separation from contained to container is zero, as uncertainty will not allow us to distinguish between them. On the other hand, there is a distinction between container and contained, as some part of the former is beyond the limits of the latter. From this point of view, our MQD calculates the distance needed to "project" one fuzzy number into the another.

Then, the MQD is used for multi-criteria decision making analysis through a weighted mean asymmetric index (WMAI). We apply this index to model subjective information on candidate products for a "pilot test" that will be carried out in a market segment. Among them, there is the need to select the one that fits better to this market and that will help collect valuable information in order to introduce the best possible product. Both, the ideal product and the candidate products, are modeled using TrFN.

The remaining of this paper is organized as follows. Section 2 briefly describes some of the newest research made on ranking of fuzzy numbers. The theoretical framework of fuzzy sets and fuzzy numbers is laid out in Section 3. Our asymmetric index is described in Section 4. Section 5 contains the application to an example. Finally, we present some conclusions in Section 6.

2. Recent work

The traditional application of indexes to evaluate the difference between fuzzy numbers has been in the process of ranking. This line of research was initiated by Jain [12], and followed by many researchers who developed algorithms with different levels of complexity. Recent works that can be mentioned are those of Asady [2,3] who worked on revisions of [20] and [4] to overcome problems that only later arose. In [8], Chou et al. worked on improvements over the widely used method developed by Chen [5], based on the utility value of a fuzzy number. Allahviranloo et al. [1] developed a method based on weighted interval-value approximations defined as crisp-set approximations of fuzzy numbers. Rezvani [16] defined a similarity measure based on the perimeter of generalized fuzzy numbers. Finally, the work more closely related to our approach is that of Nejad and

Mashinchi [15], based on the evaluation of areas between fuzzy numbers in order to rank them.

3. Fuzzy subsets and fuzzy numbers

In cases when information is subjective, models based on the theory of fuzzy subsets can help the decision maker in the evaluation of the alternatives. In this section, we will present the basic definitions of both, fuzzy sets and fuzzy numbers, that we will use throughout this paper.

Definition 1. *A fuzzy subset $\tilde{A}$ can be represented by a set of pairs composed of the elements x of the universal set X, and a grade of membership $\mu_{\tilde{A}}(x)$:*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\ ,\ \mu_{\tilde{A}}(x) \in [0,1]\}\,. \tag{1}$$

Definition 2. *An α-cut of a fuzzy subset $\tilde{A}$ is defined by:*

$$A_\alpha = \{x \in X :\ \mu_{\tilde{A}} \geq \alpha\}, \tag{2}$$

i.e., the subset of all elements that belong to $\tilde{A}$ at least in a degree α.

Definition 3. *A fuzzy subset $\tilde{A}$ is convex, iff:*

$$\lambda x_1 + (1 - \lambda x_2) \in A_\alpha\ \forall x_1, x_2 \in A_\alpha,\ \alpha, \lambda \in [0,1]\,, \tag{3}$$

i.e., all the points in $[x_1, x_2]$ must belong to A_α, for any α.

Definition 4. *A fuzzy subset $\tilde{A}$ is normal, iff:*

$$\max\left(\mu_{\tilde{A}}(x)\right) = 1,\ \forall x \in X. \tag{4}$$

Definition 5. *The core of a fuzzy subset $\tilde{A}$ is:*

$$N_{\tilde{A}} = \{x :\ \mu_{\tilde{A}}(x) = 1\}\,. \tag{5}$$

Definition 6. *A fuzzy number $\tilde{A}$ is a normal, convex fuzzy subset with domain in $\mathbb{R}$ for which:*

1. *$\bar{x} := N_{\tilde{A}}$, card $(\bar{x}) = 1$, and*
2. *$\mu_{\tilde{A}}(x)$ is at least piecewise continuous.*

The mean value [25] $\bar{x}$, also called maximum of presumption [13], identifies a fuzzy number in such a way that the proposition "about 9" can be modeled with a fuzzy number whose maximum of presumption is $x = 9$. As Zimmermann [25] explains, for computational simplicity there is a tendency to call "fuzzy number" any normal, convex fuzzy subset whose membership function is, at least, piecewise continuous, without taking into consideration the uniqueness of the maximum of presumption. Thus, this definition will include "fuzzy intervals", fuzzy numbers in which $\bar{x}$ covers an interval. As a matter of fact, Dubois and Prade [9] called them "flat fuzzy numbers".

Definition 7. *A TrFN is defined by the membership function:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-x_1}{x_2-x_1}, & \text{if } x_1 \leq x < x_2 \\ 1, & \text{if } x_2 \leq x \leq x_3 \\ \frac{x_4-x}{x_4-x_3}, & \text{if } x_3 < x \leq x_4 \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

A TrFN is represented by a 4-tuple whose first and fourth elements correspond to the extremes from where the membership function begins to grow, and whose second and third components define the interval that limits the maximum of presumption, i.e., $\tilde{A} = (x_1, x_2, x_3, x_4)$.

4. The asymmetric index

In the traditional distance functions between TrFN (based on Manhattan, Euclidean, and, in general, Minkowski's distance functions), the distance between two TrFNs $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$, when the former is totally contained in the later, is different from zero.

Nevertheless, one should study the assumption that due to uncertainty the distance from contained to container should be zero, as it would be impossible to distinguish between them. On the other hand, there is some value of distance between container and contained, as some portion of the former is outside the limits of the later.

The objective is, therefore, to calculate a mean index $D(\tilde{A}, \tilde{B})$ that shows the distance needed to make $\tilde{A} \subseteq \tilde{B}$. This function implies a sort of "projection" of $\tilde{A}$ into $\tilde{B}$ in the four regions of the set $Z = \{L_1, L_2, R_1, R_2\}$ shown in Figure 1.

Definition 8. *Given the set $\{\overline{x_1x_2}, \overline{x_2x_3}, \ldots, \overline{x_{n-1}x_n}, \overline{x_nx_1}\}$, where $\overline{x_ix_{\hat{i}}}$ is a segment defined by the points P_i and $P_{\hat{i}}$, whereas x_i and $x_{\hat{i}}$ are the abscissas of these points, we will define the region Λ as the area inscribed in the convex polygon composed by the elements of this set.*

From this definition, we can say that the regions from Z are described as (see Figures 2, 3, 4 and 5):

$$L_1 = \begin{cases} \left\{\overline{a_1b_1}, \overline{b_1x'}, \overline{x'a_1}\right\}, & \text{if } a_1 < b_1 \text{ and } a_2 > b_2, \\ \left\{\overline{a_1b_1}, \overline{b_1b_2}, \overline{b_2a_2}, \overline{a_2a_1}\right\} & \text{if } a_1 \leq b_1 \text{ and } a_2 \leq b_2, \\ \left\{\overline{a_2b_2}, \overline{b_2x'}, \overline{x'a_2}\right\}, & \text{if } a_1 > b_1 \text{ and } a_2 < b_2, \\ \emptyset, & \text{otherwise.} \end{cases} \tag{7}$$

$$R_1 = \begin{cases} \left\{\overline{b_4a_4}, \overline{a_4a_3}, \overline{a_3b_3}, \overline{b_3b_4}\right\}, & \text{if } b_3 \leq a_3 \text{ and } b_4 \leq a_4, \\ \left\{\overline{b_4a_4}, \overline{a_4x''}, \overline{x''b_4}\right\}, & \text{if } b_3 > a_3 \text{ and } b_4 < a_4, \\ \left\{\overline{b_3a_3}, \overline{a_3x''}, \overline{x''b_3}\right\}, & \text{if } b_3 < a_3 \text{ and } b_4 > a_4, \\ \emptyset, & \text{otherwise.} \end{cases} \tag{8}$$

$$R_2 = \begin{cases} \left\{\overline{b_4a_1}, \overline{a_1a_2}, \overline{a_2b_3}, \overline{b_3b_4}\right\}, & \text{if } a_1 \geq b_4, \\ \left\{\overline{b_3a_2}, \overline{a_2x'''}, \overline{x'''b_3}\right\}, & \text{if } a_1 < b_4 \text{ and } a_2 > b_3, \\ \emptyset, & \text{otherwise.} \end{cases} \tag{9}$$

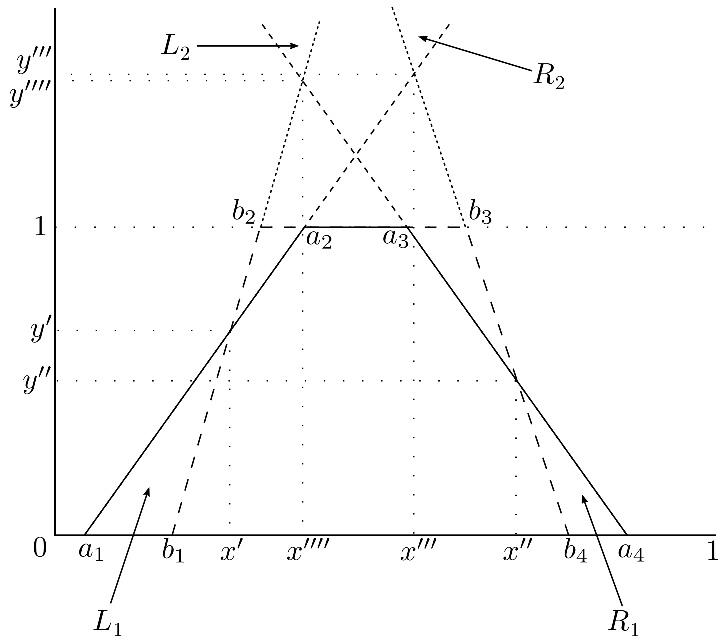

Fig. 1. Regions where the index is calculated.

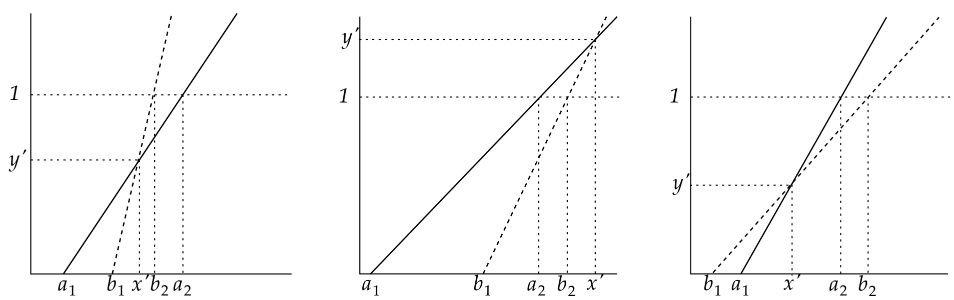

Fig. 2. Variants of the L_1 region.

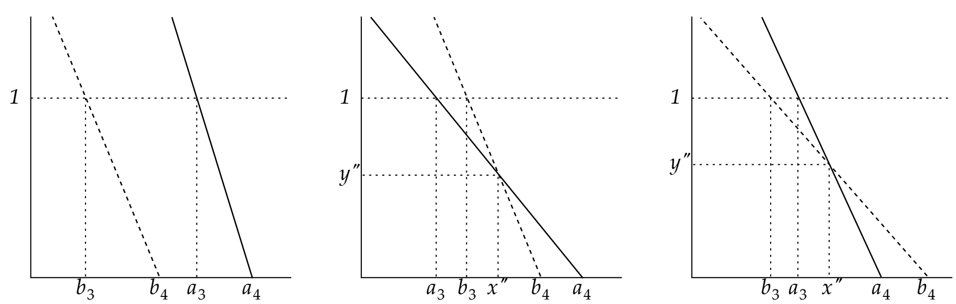

Fig. 3. Variants of the R_1 region.

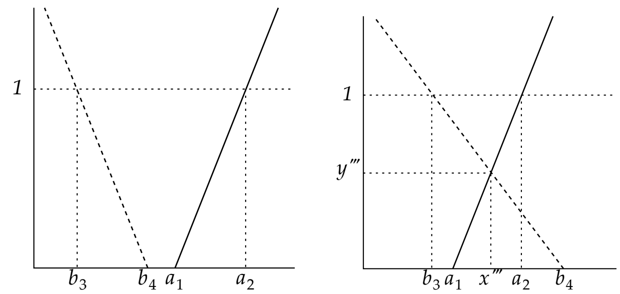

Fig. 4. Variants of the R_2 region.

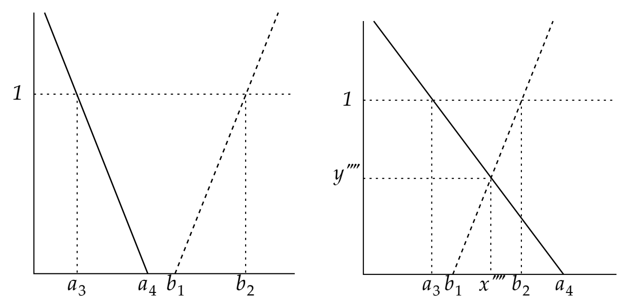

Fig. 5. Variants of the L_2 region.

$$L_2 = \begin{cases} \{\overline{a_4 b_1}, \overline{b_1 b_2}, \overline{b_2 a_3}, \overline{a_3 a_4}\}, & \text{if } a_4 \leq b_1, \\ \{\overline{a_3 b_2}, \overline{b_2 x''''}, \overline{x'''' a_3}\}, & \text{if } a_3 < b_2 \text{ and } a_4 > b_1, \\ \emptyset, & \text{otherwise.} \end{cases} \tag{10}$$

Proposition 1. *Two TrFN $\tilde{A}$ and $\tilde{B}$ intersect at maximum four points (x', y'), (x'', y''), (x''', y'''), and (x'''', y'''') such that:*

$$x' = \frac{a_1 b_2 - b_1 a_2}{a_1 - a_2 - b_1 + b_2} \; ; \; y' = \frac{a_1 - b_1}{a_1 - a_2 - b_1 + b_2}, \tag{11}$$

$$x'' = \frac{a_3 b_4 - b_3 a_4}{a_3 - a_4 - b_3 + a_4} \; ; \; y'' = \frac{b_4 - a_4}{a_3 - a_4 - b_3 + a_4}, \tag{12}$$

$$x''' = \frac{a_2 b_4 - a_1 b_3}{a_2 - a_1 + b_4 - b_3} \; ; \; y''' = \frac{b_4 - a_1}{a_2 - a_1 + b_4 - b_3}, \tag{13}$$

$$x'''' = \frac{a_4 b_2 - a_3 b_1}{a_4 - a_3 + b_2 - b_1} \; ; \; y'''' = \frac{a_4 - b_1}{a_4 - a_3 + b_2 - b_1}. \tag{14}$$

Proof. Given the equation of the straight line:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \, (x - x_1) \, , \tag{15}$$

if segments $\overline{a_1 a_2}$ and $\overline{b_1 b_2}$ intersect at (x', y'), then:

$$y' - 0 - \frac{1-0}{a_2 - a_1}(x' - a_1) = y' - 0 - \frac{1-0}{b_2 - b_1}(x' - b_1) \tag{16}$$

$$\frac{x' - a_1}{a_2 - a_1} = \frac{x' - b_1}{b_2 - b_1} \tag{17}$$

$$(b_2 - b_1 - a_2 + a_1)\, x' = -a_2 b_1 + a_1 b_1 + a_1 b_2 - a_1 b_1 \tag{18}$$

$$x' = \frac{a_1 b_2 - a_2 b_1}{a_1 - a_2 - b_1 + b_2}. \tag{19}$$

Substituting (19) on the equation of the segment $\overline{a_1 a_2}$, we solve for y':

$$y' = \frac{\frac{a_1 b_2 - a_2 b_1}{a_1 - a_2 - b_1 + b_2} - a_1}{(a_2 - a_1)} \tag{20}$$

$$= \frac{\frac{-a_2 b_1 - a_1^2 + a_1 a_2 + a_1 b_1}{a_1 - a_2 - b_1 + b_2}}{(a_2 - a_1)} \tag{21}$$

$$= \frac{(a_1 - b_1)(a_2 - a_1)}{(a_1 - a_2 - b_1 + b_2)(a_2 - a_1)} \tag{22}$$

$$= \frac{a_1 - b_1}{a_1 - a_2 - b_1 + b_2} \tag{23}$$

It is obvious that when segments $\overline{a_1 a_2}$ and $\overline{b_1 b_2}$ are parallel, they will not intersect and, thus, the point (x', y') does not exists. This demonstration is equivalent for the other intersection points.

The first step in the calculation of our index is based on a mean quadratic distance (MQD) function that measures the separation of both TrFNs in the defined regions.

Definition 9. *The MQD function between two TrFN $\tilde{A}$ and $\tilde{B}$ for each region $\zeta \in Z$ is obtained through:*

$$D_\zeta = \frac{\int_{\alpha_\zeta}^{\beta_\zeta} (b_\zeta - a_\zeta)^2 \, dy}{\beta_\zeta - \alpha_\zeta}, \tag{24}$$

where a_ζ is the equation of the line that limits ζ on the left side, b_ζ is the equation of the line that limits this region on the right side, both expressed in terms of y, and $\{\alpha_\zeta, \beta_\zeta\} \in [0,1]$, $\alpha_\zeta \le \beta_\zeta$, are the integration limits in y that we find through Figures 2, 3, 4 and 5, and Proposition 1.

It seems evident that, from Definition 9, the area of $\tilde{A}$ contained in $\tilde{B}$ generates an MQD equal to zero, like, for example, that from segment $\overline{y' a_2}$ to segment $\overline{y' b_2}$ in Figure 1.

The closed form expressions of (24) for all regions are:

$$D_{L_1} = \begin{cases} \frac{a_1^2 + a_1 a_2 + a_2^2 - 2\, a_1 b_1 + b_1^2 - a_1 b_2 + b_2^2 - a_2 b_1 + b_1 b_2 - 2\, a_2 b_2}{3}, & \text{if } a_1 \le b_1 \text{ and } a_2 \le b_2, \\ \frac{(a_1 - b_1)^2}{3}, & \text{if } a_1 < b_1 \text{ and } a_2 > b_2, \\ \frac{(a_2 - b_2)^2}{3}, & \text{if } a_1 > b_1 \text{ and } a_2 < b_2, \\ 0, & \text{otherwise.} \end{cases} \tag{25}$$

$$D_{R_1} = \begin{cases} \frac{a_3^2+a_3a_4+a_4^2-2\,a_3b_3+b_3^2-a_3b_4+b_4^2-a_4b_3-2\,a_4b_4+b_3b_4}{3}, & \text{if } b_3 \leq a_3 \text{ and } b_4 \leq a_4, \\ \frac{(a_4-b_4)^2}{3}, & \text{if } b_3 > a_3 \text{ and } b_4 < a_4, \\ \frac{(a_3-b_3)^2}{3}, & \text{if } b_3 < a_3 \text{ and } b_4 > a_4, \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

$$D_{R_2} = \begin{cases} \frac{a_1^2+a_1a_2+a_2^2-a_1b_3+b_3^2-2\,a_1b_4+b_4^2-2\,a_2b_3-a_2b_4+b_3b_4}{3}, & \text{if } a_1 \geq b_4, \\ \frac{(a_2-b_3)^2}{3}, & \text{if } a_1 < b_4 \text{ and } a_2 > b_3, \\ 0, & \text{otherwise.} \end{cases} \tag{27}$$

$$D_{L_2} = \begin{cases} \frac{a_3^2+a_3a_4+a_4^2-a_3b_1+b_1^2-2\,a_3b_2+b_2^2-a_4b_2-2\,a_4b_1+b_1b_2}{3}, & \text{if } a_4 \leq b_1, \\ \frac{(a_3-b_2)^2}{3}, & \text{if } a_3 < b_2 \text{ and } a_4 > b_1, \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

Definition 10. *The mean asymmetric index between two TrFN obtained from (25), (26), (27) and (28) is:*

$$D\left(\tilde{A}, \tilde{B}\right) = \begin{cases} \sqrt{\frac{SD}{N}}, & \text{if } N > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{29}$$

where:

$$SD = D_{L_1} + D_{L_2} + D_{R_1} + D_{R_2},$$
$$N = \mathbb{1}_{\{D_{L_1}>0\}} + \mathbb{1}_{\{D_{L_2}>0\}} + \mathbb{1}_{\{D_{R_1}>0\}} + \mathbb{1}_{\{D_{R_2}>0\}}.$$

Remark 1. It is straightforward to observe that $N = 0 \iff \tilde{A} \subset \tilde{B}$. Thus $D(\tilde{A}, \tilde{B}) = 0 \iff \tilde{A} \subset \tilde{B}$.

Remark 2. Because we would like to know the mean distance for the regions of $\tilde{A}$ not covered by $\tilde{B}$, the resulting asymmetric index sometimes behaves as a hemimetric, which implies:

1. $\exists \tilde{A} \neq \tilde{B} : D(\tilde{A}, \tilde{B}) = 0$. Again, we would be saying that $\tilde{A} \subset \tilde{B} \iff D(\tilde{A}, \tilde{B}) = 0$.
2. $\exists \tilde{A} \neq \tilde{B} : D(\tilde{A}, \tilde{B}) \neq D(\tilde{B}, \tilde{A})$. As a matter of fact, there are two cases where $D(\tilde{A}, \tilde{B}) = D(\tilde{B}, \tilde{A})$. Firstly, if for a given point:

$$b_1 \in \mathbb{R}, B = (b_1, b_1 + a_4 - a_3, b_1 + a_4 - a_2, b_1 + a_4 - a_1), \tag{30}$$

i.e., when $\tilde{A}$ has the inverse shape of $\tilde{B}$. Secondly, if $\tilde{A} = \tilde{B} + c$, with $c \in \mathbb{R}$, i.e., when $\tilde{A}$ and $\tilde{B}$ have the same shape.

The mean asymmetric index based on the MQD function models the separation between the assessment $\tilde{P}_i$ given to some candidate resource P, in any given characteristic i, and the required value $\tilde{I}_i$ asked from an "ideal" candidate I in that same characteristic. We will now define the weighted mean asymmetric index (WMAI) between all the characteristics evaluated in a candidate and the required levels of those characteristics.

Definition 11. *The weighted mean asymmetric index (WMAI) between $\tilde{P} = \{\tilde{P}_1, \tilde{P}_2, \ldots, \tilde{P}_n\}$ and $\tilde{I} = \{\tilde{I}_1, \tilde{I}_2, \ldots, \tilde{I}_n\}$ will be:*

$$\delta(P, I) = \sum_{i=1}^{n} \omega_i \cdot D\left(\tilde{P}_i, \tilde{I}_i\right), \tag{31}$$

where $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ is a vector of weights such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \neq 0$.

5. Application of the WMAI to the selection of a product for a pilot test

In order to illustrate the application of the methodology, we will use an example based on the introduction of a dairy product into the market.

5.1. Description of the target market segment

The first thing that needs to be defined is a mathematical descriptor that numerically and accurately reflects the market segment that the Marketing Department is interested in reaching. A hypothetical dairy company wants to introduce a new product for the market segment defined by the following characteristics:

1. Health-conscious consumer.
2. With an age above 50 years.
3. Willing to pay a high price for healthier diary product.
4. Interested in new technology driven products, but not a consumer totally devoted to technology. \item Looking for a product that can be carried around and consumed at any time.

Given this information, the marketing department models the target segment with the fuzzy set $\tilde{S} = \tilde{s}_i$, as we can see in Table 1.

Table 1. Target segment modeled as a fuzzy set.

	c_1	c_2	c_3	c_4	c_5
$\tilde{S}$=	(0.7,1)	(0.6,1)	(0.5,0.8,1)	(0.7,0.8,0.9)	(0.8,1)

Each fuzzy number $\tilde{s}_i$ represents an assessment of the ideal level that the target segment has on each one of the characteristics in the set $C = c_i$. Both, the characteristics and the meaning of the assessments given to this particular segment, are defined as:

1. c_1- Health-consciousness: A consumer is regarded as health-conscious if at least 70% of his food purchases are done taking health into consideration.
2. c_2- Maturity: A consumer is considered mature if he has already lived more than 60% of his life expectancy.
3. c_3- Price level: In this case, the consumer prefers to buy products in the top 50% of the price scale, with a maximum preference for products below 80%.
4. c_4 - Novelty of organoleptic or technological characteristics: the consumer prefers to buy a new product between 70 and 90% of the times, with a maximum of 80%, if new flavors or properties are also involved.
5. c_5 - Easiness of transportation and consumption: when a consumer buys a product he prefers the top 80% in easiness of transportation and consumption, meaning he wants a product that can be carried around with no worries of spillage or spoilage, and that needs almost no additional procedures beyond opening its package for its consumption.

The characteristics will be weighted according to Table 2.

Table 2. Weights for the characteristics.

	c_1	c_2	c_3	c_4	c_5
ω=	0.1	0.35	0.25	0.25	0.05

5.2. Description of the candidates for the pilot test

The R&D department of the company has decided that the pilot test should be run with one of the following five products from the set $\mathfrak{P} = \{\tilde{P}^{(j)}\}, j = 1, \ldots, 5$:

1. $\tilde{P}^{(1)}$- Fructose sweetened soy yogurt with fruits: It has been assessed as a product with healthy properties above the average, without a defined age group, an average price, no real novelty in flavor or technology, and with the same limitations in transportability and consumption as most dairy products.
2. $\tilde{P}^{(2)}$- Inulin sweetened Greek yogurt enriched with calcium: Being a Greek yogurt means this is a fatty product, although this is compensated with the substitution of complex sugars with inulin, so this product is considered to be middle-of-the-pack in healthiness, although with some uncertainty. Nonetheless, by enriching it with calcium, the targeted age group is certainly mature. Price, as well as its technological appeal due to the novelty of inulin, ranges in the middle upper echelon. Of course, a Greek yogurt is a product to keep refrigerated and eaten with a spoon, which limits its transportability.
3. $\tilde{P}^{(3)}$- Aspartame sweetened, fat free chocolate pudding enriched with Omega-3: By removing fat while adding Omega-3 and aspartame, this product can be considered remarkably healthy. Age groups from young adulthood to elderly will be equally attracted to consume it. The price is above average, but not the most expensive. However, this is not a technologically driven product as all its characteristics are found in

many products. This product has the same problems of transportability described for the Greek yogurt.

4. $\tilde{P}^{(4)}$- Digestion helping, cucumber yogurt soup with *Lactobacillus casei*: A yogurt soup that is designed to aid digestion is regarded as an almost perfect element of a healthy diet. Age groups for this product go from middle-aged people to consumers entering maturity, as this flavor is not a favorite of younger groups and the introduction of external bacteria might cause problems to older groups. Price is at the top of the line, as this is considered a gourmet food. Even if, technologically speaking, there is nothing new in this product, its flavor and concept are novel enough to put this product above average in preferences of people looking for new products with new flavors. Finally, easy consumption is not quite feasible with a soup.
5. $\tilde{P}^{(5)}$- Energy boosting, tropical fruits flavored smoothie, enriched with amino acids and taurine: This product might only be considered healthy in the group of people that have an active night life, as well as for those that practice sports. This makes it more suitable for age groups ranging from young adulthood to early maturity. It is in the most expensive level, has a high impact on people looking for technology driven foods and is the easiest product to use, although it is recommended to consume it cold.

Thus, each $\tilde{P}^{(j)} = \{\tilde{\mu}_{i,j}\}$ is a fuzzy set with the same number of elements as $\tilde{S}$, modeled according to the information gathered as shown in Table 3.

Table 3. Assessments made by the company experts on the new products.

	c_1	c_2	c_3	c_4	c_5
$\tilde{P}^{(1)}$=	(0.5,0.6,0.6,0.7)	(0.1,0.1,1,1)	(0.5,0.5,0.5,0.5)	(0,0,0.1,0.1)	(0.5,0.5,0.5,0.5)
$\tilde{P}^{(2)}$=	(0.4,0.5,0.5,0.6)	(0.7,0.7,1,1)	(0.7,0.7,0.8,0.8)	(0.7,0.7,0.9,0.9)	(0.4,0.4,0.4,0.4)
$\tilde{P}^{(3)}$=	(0.8,0.8,1,1)	(0.3,0.3,1,1)	(0.8,0.8,0.8,0.8)	(0.7,0.7,0.7,0.7)	(0.4,0.4,0.4,0.4)
$\tilde{P}^{(4)}$=	(0.8,0.8,0.9,0.9)	(0.4,0.4,0.7,0.7)	(1,1,1,1)	(0.5,0.5,0.7,0.7)	(0.1,0.1,0.1,0.1)
$\tilde{P}^{(5)}$=	(0.2,0.2,0.4,0.4)	(0.3,0.3,0.7,0.7)	(1,1,1,1)	(1,1,1,1)	(0.9,0.9,0.9,0.9)

5.3. Results

We now proceed to calculate the distance between each one of the new products proposed for a test run and the target segment. The product closest to the target segment will be the one selected for this test.

$$\delta\left(\tilde{P}^{(j)}, \tilde{S}\right) = \sum_{i=1}^{5} \omega_i \, D\left(\tilde{\mu}_{i,j}, \tilde{s}_i\right)$$

$$\begin{aligned}\delta\left(\tilde{P}^{(1)},\tilde{S}\right) &= 0.1\cdot 0.12+0.35\cdot 0.5+0.25\cdot 0.17+0.25\cdot 0.7+0.05\cdot 0.3\\ &= 0.42.\end{aligned}$$

$$\begin{aligned}\delta\big(\tilde{P}^{(2)},\tilde{S}\big) &= 0.1\cdot 0.21+0.35\cdot 0+0.25\cdot 0.06+0.25\cdot 0.06+0.05\cdot 0.4\\ &= 0.07.\end{aligned}$$

$$\begin{aligned}\delta\big(\tilde{P}^{(3)},\tilde{S}\big) &= 0.1\cdot 0+0.35\cdot 0.3+0.25\cdot 0+0.25\cdot 0.06+0.05\cdot 0.4\\ &= 0.14.\end{aligned}$$

$$\begin{aligned}\delta\big(\tilde{P}^{(4)},\tilde{S}\big) &= 0.1\cdot 0+0.35\cdot 0.2+0.25\cdot 0.12+0.25\cdot 0.18+0.05\cdot 0.7\\ &= 0.18.\end{aligned}$$

$$\begin{aligned}\delta\big(\tilde{P}^{(5)},\tilde{S}\big) &= 0.1\cdot 0.41+0.35\cdot 0.3+0.25\cdot 0.12+0.25\cdot 0.15+0.05\cdot 0\\ &= 0.21.\end{aligned}$$

As we can see, the product closest to the market segment targeted with our pilot test is the Greek yogurt, a product that meets most of the requirements even if transportability is not a distinguished feature. The chocolate pudding is the closest competitor to the Greek yogurt, but seems that it would need a strong marketing campaign to introduce it in the age group of the target segment. The worst suited is the soy yogurt, maybe because it is a generic product that can be used as a baseline, to check how well targeted are other products.

We can present the results in terms of preference using the precedence operator:

$$\tilde{P}^{(1)} \prec \tilde{P}^{(5)} \prec \tilde{P}^{(4)} \prec \tilde{P}^{(3)} \prec \tilde{P}^{(2)}.$$

This means that the Greek yogurt is the product best suited for a test run, then the chocolate pudding, the cucumber yogurt soup, the energy boosting smoothie, and the soy yogurt, respectively.

6. Conclusions

In this work, we presented a mean quadratic distance (MQD) function that calculates the distance needed to have one TrFN contained by another. This MQD function generates a distance of magnitude zero for the areas of the first TrFN overlapped by the second, while considering distances bigger than zero for those non-overlapped.

By calculating the MQD over these non-overlapped regions, we obtain a weighted mean asymmetric index (WMAI) of separation between two vectors of TrFN. The WMAI, calculated from assessments given over a set of characteristic of a group of candidates to the “ideal” requirements on those characteristics, is then obtained and used as the decision variable.

This methodology is applied to the evaluation of new products that are candidates for a test run in a particular target segment. It allows the comparison of products with different features, some close to the target segment and some others far from it, to take a decision accordingly. A product test run might be an expensive affair, and as such, the decision on which product use for it has to be as fully supported as possibly.

A good feature of this methodology is that statistical information, adequately transformed or processed by experts, can be used together with subjective information, usually disregarded by statistical methods, to get a more comprehensive view of the situation and give better help to the decision maker.

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