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Stability of Beta Coefficients of Sector and Subsector Portfolios in an Uncertain Environment

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Abstract. This paper is a first approach to the study of beta coefficients using fuzzy regression. We intend to improve the calculation of the sector and subsector betas of the Spanish Stock Market using fuzzy regression in an attempt to incorporate all future inaccuracies and the subjectivity associated with decision making. Our objective is to use all the information provided by the market to determine the systematic risk.

Keywords: risk, beta coefficient, fuzzy regression, CAPM.

1. Introduction

As is well known, the Capital Asset Pricing Model (CAPM) is a model for establishing optimal portfolios. It sets the expected return on any asset as a positive linear function of its systematic risk measured by means of the beta coefficient (β). This concept emphasizes the importance of systematic risk as a measure of non-diversifiable risk, the only risk that is remunerated in financial markets.

Since betas are non-observable, approximations that are typically based on historical data must be used. The basic underlying notion of this model is that every asset is affected by the market's general movements, assuming that the market factor is a systematic force. Other effects are assumed to be specific or unique to an individual asset and they diversify in a portfolio. One measure of the response of assets to changes in the markets could be obtained by relating the asset performance, R_j, to the performance of the market index, R_M, according to the following expression:

$$Rjt=aj+\beta j\ RMt+\varepsilon jt \quad j=1,2,\ldots,N; t=1,\ldots,T$$

In practice, beta is the ordinary least squares estimator (OLS) of the return on asset j on the portfolio return over a period of time. This estimation, besides using historical returns, requires other practical assumptions. Each assumption can significantly affect results.

The aim of this study is to determine what impact the size of the portfolio and periodicity of the data have on the stability of betas. To do so we shall use fuzzy regression, taking into account all the stock quotes information during the trading days. Results are consistent with the literature but, nevertheless, there is an excess of information. Data was taken from Madrid´s stock exchange in the period 2005–2009, sector and subsector portfolios were studied and Tanaka & Ishibuchi's model was used (see [37]).

This paper begins by discussing related work. In Section 3 we present a brief description of the Tanaka e Ishibuchi fuzzy regression model that we will use later. In Section 4, we will use fuzzy regression to estimate the sector and subsector beta from the Spanish Stock Market in Madrid and analyze obtained data. Finally, we will present the findings.

2. Related Work

An essential requirement for using beta to obtain the future risk of a financial asset is that it has predictive power. Since future values are calculated from past data, they must be stable over time so that the estimation is correct and precise. Therefore, the more stable a value is over time, the more useful it will be. Although beta is an indicator of risk, its value is not unique and its result will depend on the hypothesis and data that are used. Many authors have studied beta´s historical evolution, and analyzed its capacity to make predictions from empirical and theoretical points of view.

The first decision that must be considered when calculating betas is the length of the sample period. A longer period provides more data, but the company itself could have changed its risk characteristics.

A conceptual problem arises when we try to determine the return on an asset. Financial theory does not specify if returns should be considered on a daily, weekly or even monthly basis. Several studies have shown that beta coefficients can vary substantially depending on the possession period by which their performances have been determined. The magnitude of such changes provides a measure of beta stability. Moreover, the calculation of betas will depend on which price is considered: closing price, average daily price, etc.

Various studies [1], [2], [3], [4], [5], [6] analyze the relation between the length of the estimation period and beta stationarity. They find that the prediction ability of betas (and consequently their stationarity) increases with the length of the period. However, this increase decreases in more diversified portfolios.

Beta assets vary from one period to another because, in the first place, the risk measured by the beta coefficient of a value can vary over time. In the second place, each period's beta is calculated with a random error which increases as the coefficient goodness and the prediction power decrease. If we consider a portfolio, random errors committed in the calculation of individual betas will tend to cancel each other out, so a portfolio beta is more stable than a single beta value.

Two studies, [1] and [2], analyze the seasonality of betas of individual securities and portfolios. They observe that, whereas betas of portfolios with a high number of securities provide a considerable amount of information about future betas, the betas of individual securities provide much less. This result suggests that a portfolio's beta is more stable than a single security's beta. The same direct relationship between the portfolio size and the beta stationarity has been observed in various studies [3], [4], [7], [8]. Another study [9] states that sector betas vary very little and, therefore, recommends using the calculated beta of one sector.

Traditional studies on the stability of portfolio betas differ from each other mainly in two areas: the portfolio construction method and the stability test.

Some of the traditional studies mentioned above have used the same portfolio construction method. This involves classifying portfolio securities according to their historical beta. In this way, they produce portfolios of N assets each. The assets with the highest beta are assigned to the first, the assets with the next highest beta to the second and so on until the smallest N is included in the last beta. This procedure is questioned by [10], which attributes the results of [1] and [2] to the portfolio selection method. If it had been random, there would have been no significant increase in the stationarity of the portfolio beta, even when it increased in size. They conclude, in short, that the results of traditional tests are a direct result of the portfolio construction method, and not of the increase in securities. However, [11] shows that both methods are valid and, therefore, that they lead to consistent results.

The most plausible explanation for the results obtained by [10] is the combination in this study of the random method of portfolio construction and the particular stability test used. This test was also used in pioneering stability work and involves calculating the portfolio betas for every two consecutive assessment periods and obtaining the correlation coefficients between them. If these coefficients in the different periods have high values (close to 1), the betas would be significantly stable. Otherwise, they would not be.

As well as indicating to what extent beta values change over time, this procedure makes it possible to detect the extent to which betas remain in the same group in successive time periods (see [4]). Like portfolios constructed on the basis of securities ranked by their beta, it will be difficult to produce changes in the beta value that are big enough to make them change their risk class. In fact, many studies have shown that high or low portfolio betas are more stable than intermediate portfolio betas. Instead, with randomly grouped securities, portfolios change their risk class more often.

In short, it is reasonable to expect correlation coefficients to be higher with prior holdings of securities ranked by their beta than with randomly selected securities portfolios. This explains the results obtained by [10] and the observation made by [11] that beta stabilities improved, regardless of the portfolio construction method employed, when the stability test used was the calculation of absolute deviations in betas rather than correlation coefficients. Using the mean absolute deviation as a measure of beta stationarity, these studies observed that it decreased as the number of securities in the portfolio increased.

In the light of the above, and in order to measure the instability of betas not the risk classes, [7] considers that it is much more appropriate to construct portfolios with randomly selected titles and some measure of deviation or change in those values over time, rather than correlation coefficients.

[7] (p. 46) stresses that “in the real world, investors are more worried that their portfolios do not change their risk class than they are of the changes in the order of their

portfolios in relation to other portfolios. Seasonality, in this way, should be an absolute measure and not a relative one”. For this reason, the author proposes the simultaneous satisfaction of two conditions so that it can be said that beta is stationary. First, historical or ex-post betas should be an adequate approximation of future or ex-ante betas. This condition must be fulfilled if betas are to be used for predictive purposes. Second, the value of the future beta must not exceed certain limits that are acceptable to investors, so that the portfolio can remain within the same risk class on the considered horizon. This condition will be satisfied if the standard deviation (or variance) of the ex-ante beta is small, as this will mean that expected beta values have low dispersion around an expected mean value.

In short, stationarity improves when the number of portfolio securities increases if the average ex-post beta provides a better approximation of the average ex-ante beta as the size of the portfolio increases and if the standard deviation of the ex-ante beta decreases when the number of securities in the portfolio increases.

Another line of analysis is the hypothesis that betas vary over time. In [12] a conditional CAPM is specified, on the basis that the beta and expected returns vary over time. The results are better than those of the static model. Similarly, [13] uses 6 different techniques to make a study of 18 sectors in Europe, and shows that variable betas estimate the profitability of the sector, explained in terms of market movements, more efficiently than OLS. Similar results can be found in [14] and [15].

The previous empirical and theoretical literature on factors that can influence beta stability usually focused on a risk environment. This perspective highlights the instability of betas. In an attempt to incorporate all the underlying future uncertainty and the subjectivity related to the decision making process, we propose a further step that uses elements of the Theory of Fuzzy Sets. In particular, we propose to estimate the market model using fuzzy regression methods.

The objective of fuzzy regression is to determine a functional relationship between a dependent variable and a set of independent variables. As we will show, fuzzy regression is in many ways more versatile than conventional linear regression because functional relationships can be obtained when the independent variables, dependent variables, or both, are not crisp values but intervals or fuzzy numbers.

In contrast to ordinary regression, which is based on probability theory, fuzzy regression can be based on possibility theory and fuzzy set theory. In ordinary regression analysis, the unfitted errors between a regression model and observed data are generally assumed to be observation error, which is a random variable with a normal distribution, constant variance, and a zero mean. In fuzzy regression analysis, the same unfitted errors are viewed as the fuzziness of the model structure, as was initially developed in [16]. Subsequently, [17], [18], [19], [20], [21], [22], [23], [24], [25], [26] and [27] made other contributions by applying different optimization criteria for a linear or curved adjustment. The literature on fuzzy regression applied to finance is growing. Some of the most recent contributions are [28], [29], [30], [31], [32], [33] and [34].

This modelling technique has some advantages over the traditional regression technique. It enables all the available information on prices to be incorporated. It is not limited to a single or an average price. In financial markets the same asset is traded at different prices during market hours. When econometric techniques are used, a single number must quantify observations (closing prices, average prices, etc.). In this process, a great deal of information is lost. The selection of one value or another is arbitrary. Fuzzy regression methods, on the other hand, make it possible to adjust the functional relation using all the information available about the observed values. In addition, the results of estimations are fuzzy numbers, not random variables, so they are simpler to treat and less demanding in terms of assumptions. For a more rigorous analysis of this issue see [35] and [36].

3. Fuzzy Regression using the Tanaka and Ishibuchi Model

The goal of fuzzy regression is to determine a functional relation between a dependent variable and several explanatory variables, where the estimated parameters are confidence intervals (CI). For a more rigorous analysis of this issue, see [35], [36], [38] and [39].

A CI A is represented by its upper and lower bounds as $A=[a_1, a_2]$; or by its centre and its radius $A=\langle a_C, a_R\rangle$ where:

$$a_C = (a_1 + a_2)/2 \qquad a_R = (a_1 - a_2)/2$$

If we have a sample $\{(Y_1, X_1), (Y_2, X_2), \dots, (Y_j, X_j), \dots, (Y_n, X_n)\}$ where:

- Y_j is the j-th observation of the dependent variable, j=1,2,…,n, expressed by a confidence interval $Y_j = [Y_j^1, Y_j^2] = \langle Y_{jC}, Y_{jR}\rangle$
- X_j is the vector of the j-th observation of the independent variables, with j=1,2,…,n. Then, X_j is an m-dimensional variable $X_j = (X_{0j}, X_{1j}, X_{2j}, \dots, X_{ij}, \dots, X_{mj})$ where $X_{0j}=1\ \forall j$, and X_{ij} is the value of the j-th observation for the i-th variable. We assume that observations are crisp.

The relation between the dependent and independent variables is linear:

$$Y = A_0 + A_1 X_1 + A_2 X_2 + \cdots + A_m X_m$$

where A_i, i =0,1,…,m are CI:

$$A_i = \langle a_{iC}, a_{iR}\rangle \quad i = 0, 1, \dots, m$$

The final goal is to determine centres and radiuses of A_i that are compatible with the available observations.

In order to estimate the value of the j-th independent variable, $\hat{Y}_i = \langle \hat{Y}_{iC}, \hat{Y}_{iR}\rangle$, we do the sum:

$$\langle \hat{Y}_{iC}, \hat{Y}_{iR}\rangle = \sum \langle a_{iC}, a_{iR}\rangle X_{ii} = \langle \sum a_{iC} X_{ii}, \sum a_{iR} |X_{ii}|\rangle, \quad j = 1, 2, \dots, n$$

The goodness of fit is inversely related to the uncertainty (width) of the estimations of the observations $Y_i, \hat{Y}_i$. The width of $\hat{Y}_i$ is the radius of the confidence interval $\hat{Y}_{iR}$, which is obtained in the following way:

$$\hat{Y}_{iR} = \sum a_{iR} |X_{ii}| = a_{0R} + a_{1R}|X_{1i}| + \cdots + a_{mR}|X_{mi}|$$

Then, the total uncertainty of all the sample estimations, z, is the sum of the radiuses of the estimations:

$$z = \sum \hat{Y}_{iR} = \sum \sum a_{iR}|X_{ii}|$$

The parameters A_i must achieve not only the least possible uncertainty of $\hat{Y}_j$, but also that $\hat{Y}_j$, be as close as possible to the observation of the explained variable Y_j. In this context, we define two approximations of $\hat{Y}_j$ congruent with Y_j. [37] postulates that the observation must be included within its estimation: $Y_j \subseteq \hat{Y}_j \; \forall j$. In other words:

$$Y_{iC} - Y_{iR} \geq \hat{Y}_{iC} - \hat{Y}_{iR}$$
and
$$Y_{iC} + Y_{iR} \leq \hat{Y}_{iC} + \hat{Y}_{iR}$$

In order to determine the parameters, A_i, the following linear program must be solved:

$$\text{Min } z = \sum \hat{Y}_{iR} = \sum \sum a_{iR}|X_{ii}|$$
s. t:
$$\hat{Y}_{iC} - \hat{Y}_{iR} = \sum a_{iC} X_{ii} - \sum a_{iR}|X_{ii}| \leq Y_{iC} - Y_{iR} \quad j = 1, \ldots, n$$
$$\hat{Y}_{iC} + \hat{Y}_{iR} = \sum a_{iC} X_{ii} + \sum a_{iR}|X_{ii}| \geq Y_{iC} + Y_{iR} \quad j = 1, \ldots, n$$
$$a_{iR} \geq 0 \quad i = 0, 1, \ldots, m$$

The first restriction ensures that the lower bounds of the estimations are lower than the lower bounds of the observations. The second restriction guarantees that the upper bounds of the estimations are higher than the upper bounds of the observations. The third restriction imposes a positive radius for the CI.

4. Estimation of Sector and Subsector Betas

In order to perform our study, we took daily values from the General Index of the Madrid Stock Exchange (IGBM) and from the sector and subsector indices.

The specification and rationale for the temporal dimension of the rate of return is sometimes forgotten. Some studies use daily performance rates, while others use weekly, monthly or annual rates. There are, however, theoretical grounds for believing that the historical rate must be calculated from weekly quotes. First, [40] concludes that the beta coefficient estimated from weekly data is a response to the concept of instantaneous systematic risk of the market model.

Secondly, the use of daily quotes presents the problem of asynchronous or infrequent trading ([41], [42], among others), whereby the stock dynamics is not identical for all securities. This lack of synchronization in stock movements makes it advisable to space out the frequency of observations.

Thirdly, [43] argues that the week can be considered as the possession horizon for the investor for reasons of imperfection in information processing. The detection of the "weekend effect" or the "day-of-the–week effect" has led to proposals that investors should have a weekly possession horizon and objective considerations whose frequency is weekly (ordinary press releases, financial newsletters and securities, etc.).

Market return is proxied by the closing prices of the General Index of the Madrid Stock Exchange (IGBM). According to [31], in order to calculate betas using fuzzy regression models, $\left[\beta_1 , \beta_2\right]$, we express weekly returns by means of a confidence interval, $[R_1, R_2]$, whose bounds are given by:

– the lowest return, R_1, that the investor can achieve. This happens if the investor buys the asset at its highest price on day t-1 ($P_{max, t-1}$) and sells the asset on day t at the lowest price ($P_{min, t}$):

$$R_1 = (P_{min, t} - P_{max, t-1}) / P_{max, t-1}$$

– the highest return, R_2. The investor obtains this return if he buys the asset at the lowest price and sells it at the highest price:

$$R_2 = (P_{max, t} - P_{min, t-1}) / P_{min, t-1}$$

This way of calculating returns enables all the information contained in the different prices of each trading day to be included. With these data, we proceed with the estimation of the market model by using the Tanaka and Ishibuchi fuzzy regression model for the period between 01-01-2005 and 06-31-2009.

In order to verify if the number of securities and the length of the holding period influence the stability of the beta coefficient, as reported in studies with traditional techniques, we make estimations using fuzzy regression. Results are shown in Appendix A and B. Each table shows the result for a sector covered by IGBM and includes the result of the subsectors into which they are divided. So for each sector and subsector the table gives the minimum and maximum value (β_1, β_2) of the resulting interval. To analyze the importance of the length of time, the results are computed using quarterly (Table 3 to 7) and biannual data (Table 8 to 14).

According to [31] in order to verify if the number of securities and the length of the holding period influence the stability of the beta coefficient as reported in studies with traditional techniques, we use the standard deviation of the estimation of calculated betas as a comparison measure. We calculate the deviation of the lower betas, σ_{β_1}, upper betas, σ_{β_2}, the sum of both $\sigma_{\beta_1} + \sigma_{\beta_2}$, and the joint deviation of the β_1 and β_2, σ_{β_1,β_2}. The results for quarterly (biannual) betas are presented in Table 1 (Table 2).

Table 1. Quarterly beta coefficients of sector and subsector portfolios

Sector	Subsector	β average	σ_{B_1}	σ_{B_2}	$\sigma_{B_1}+\sigma_{B_2}$	σ_{B_1,B_2}
Oil And Energy		[0.82, 0.93]	0.51	0.47	0.98	0.49
	Oil	[0.58, 0.76]	1.16	0.95	2.12	1.05
	Electricity and Gas	[0.61, 0.70]	0.61	0.72	1.33	0.66
	Water and Others	[0.67, 0.76]	0.95	0.93	1.88	0.93
Basic Mat., Industry and Construction		[0.99, 1.00]	0.39	0.39	0.78	0.38
	Mineral/Metals/Transf	[0.66, 1.00]	0.49	1.51	2.00	1.12
	Capital Goods	[0.76, 0.80]	0.57	0.61	1.18	0.58
	Construction	[0.97, 1.00]	0.72	0.71	1.43	0.71
	Construct. Materials	[0.93, 0.97]	0.95	0.98	1.93	0.95
	Chemical	[0.78, 1.28]	1.46	1.52	2.98	1.49
	Engineering and Others	[0.97, 1.01]	0.73	0.84	1.57	0.78
	Aerospace	[0.58, 0.70]	1.22	1.02	2.24	1.11
Consumer Goods		[0.54, 0.63]	0.27	0.29	0.56	0.28
	Food and Beverages	[0.57, 0.59]	0.42	0.47	0.89	0.44
	Textiles/Clothing/Shoes	[0.63, 0.83]	0.55	0.66	1.21	0.61
	Paper and Graphic Arts	[0.59, 0.68]	0.63	0.47	1.10	0.55
	Other Consumer Goods	[0.03, 0.45]	0.66	0.93	1.59	0.82
	Pharmacy Products	[0.68, 0.79]	0.88	0.78	1.67	0.83
Consumer Services		[0.79, 0.82]	0.29	0.32	0.61	0.30
	Leisure/Tourism/Hotel	[0.85, 0.99]	0.62	0.84	1.46	0.73
	Retailing	[1.07, 1.19]	1.11	1.07	2.18	1.08
	Communicaction and Publicity	[0.79, 0.83]	0.62	0.68	1.30	0.64
	Car Parks/Motorways	[0.76, 0.84]	0.50	0.46	0.96	0.48
	Transport/Distribution	[0.88, 1.07]	0.81	0.63	1.44	0.72
	Other Services	[0.55, 0.62]	0.91	0.82	1.73	0.85
Financial Serv. & Real Estate		[0.93, 1.11]	0.33	0.41	0.74	0.38
	Banks	[0.92, 1.13]	0.34	0.45	0.79	0.41
	Insurance	[0.82, 0.96]	0.37	0.52	0.89	0.45
	Portfolio and Holding	[0.88, 0.98]	0.51	0.56	1.07	0.53
	Real Estate and Others	[0.47, 0.47]	0.78	0.78	1.56	0.77
Technology & Telecommunications		[0.65, 0.87]	0.50	0.85	1.35	0.70
	Electronics/Software	[0.85, 0.92]	0.51	0.54	1.05	0.52
	Telecommunications & Others	[0.64, 0.86]	0.52	0.87	1.39	0.72

Table 2. Biannual beta coefficients of sector and subsector portfolios

Sector	Subsector	β average	σ_{B_1}	σ_{B_2}	$\sigma_{B_1}+\sigma_{B_2}$	σ_{B_1,B_2}
Petrol And Power		[1,16, 1,16]	0.31	0.31	0.62	0.30
	Oil	[0.48, 0.48]	0.76	0.76	1.53	0.74
	Electricity and Gas	[0.93, 0.93]	0.74	0.74	1.48	0.72
	Water and Others	[0.83, 0.83]	0.83	0.83	1.67	0.81
Basic Mat., Industry And Construction		[1,01, 1,01]	0.38	0.38	0.76	0.37
	Mineral/Metals/Transf	[0.91, 0.98]	0.68	0.77	1.45	0.71
	Capital Goods	[0.57, 0.57]	0.52	0.52	1.04	0.51
	Construction	[0.67, 0.67]	0.90	0,90	1,81	0,88
	Construct. Materials	[0,67, 0,67]	0,90	0,90	1,81	0,88
	Chemical	[1,17, 1,17]	1,58	1,58	3,16	1,53
	Engineering and Others	[0.95, 0.95]	0.82	0.82	1.65	0.80
	Aerospace	[0.51, 0.51]	1.11	1.11	2.22	1.08
Consumer Goods		[0.61, 0.61]	0.22	0.22	0.44	0.21
	Food and Beverages	[0.52, 0.52]	0.30	0.30	0.61	0.29
	Textiles/Clothing/Shoes	[0.70, 0.70]	0.51	0.51	1.03	0.50
	Paper and Graphic Arts	[0.77, 0.77]	0.46	0.46	0.93	0.45
	Other Consumer Goods	[0.13, 0.13]	0.51	0.51	1.01	0.49
	Pharmacy Products	[0.73, 0.73]	0.67	0.67	1.34	0.65
Consumer Services		[0.92, 0.92]	0.24	0.24	0.48	0.23
	Leisure/Tourism/Hotel	[0.69, 0.69]	0.34	0.34	0.69	0.33
	Retailing	[1.41, 1.41]	1.04	1.04	2.07	1.00
	Communicaction and Publicity	[0.66, 0.66]	0.23	0.23	0.46	0.22
	Car Parks/Motorways	[1.09, 1.09]	0.46	0.46	0.92	0.45
	Transport/Distribution	[0.87, 0.87]	0.55	0.55	1.10	0.53
	Other Services	[0.74, 0.76]	0.88	0.87	1.75	0.85
Financial Serv. & Real Estate		[1.01, 1.01]	0.29	0.29	0.58	0.28
	Banks	[1.04, 1.04]	0.31	0.31	0.62	0.30
	Insurance	[1.21, 1.21]	0.49	0.49	0.98	0.48
	Portfolio and Holding	[0.85, 0.85]	0.51	0.51	1.03	0.50
	Real Estate and Others	[0.84, 0.84]	0.48	0.48	0.95	0.46
Technology & Telecommunications		[0.89, 0.89]	0.60	0.60	1.19	0.58
	Electronics/Software	[0.74, 0.74]	0.36	0.36	0.72	0.35
	Telecommunications & Others	[0.90, 0.90]	0.63	0.63	1.27	0.61

In this way, we verify that all quarterly sector betas are much more stable than those from their corresponding subsector. Similar conclusions are drawn from biannual betas.

To study the extent to which the length of the estimation period affects beta stability, portfolio betas are estimated for different intervals but the same holding period (weekly). An analysis of the beta's standard deviations for different estimation periods shows that the longer the period is the greater the stability. We verify that, in the Spanish market, the biannual beta is more stable than the quarterly beta using both ways of measuring deviation.

Some sectors are more stable than others, independently of the estimation period. Consumer Goods and Consumer Services are the most stable sectors, whereas Technology and Communications is a highly unstable sector.

Our results, obtained using a fuzzy methodology, are consistent with results of studies based on traditional econometric techniques.

5. Conclusions

In recent decades many academic studies have questioned beta stability. While earlier studies were based on simple methodologies, the development of models, algorithms and computational systems has led to more sophisticated testing techniques. Nevertheless, all these contributions take place within a risk environment. We consider that decision making processes, especially those using betas as a risk measure, take place in an uncertain environment. Therefore, in this paper we made a preliminary approach to studying this coefficient stability by using fuzzy regression models.

This modelling approach has some advantages over the traditional regression technique. In the first place, the estimations that we obtain after adjusting the fuzzy coefficients are not random variables, which are often difficult to treat numerically, but fuzzy numbers, which are easier to treat.

The fuzzy regression method presents different results from ordinary regression. The differences between fuzzy regression and ordinary regression are due to the different meanings of the deviations between the observed values and estimated values. In ordinary regression, deviations are viewed as random errors due to observation inconsistency. In fuzzy regression, the deviations are viewed as fuzzy errors due to system fuzziness. In ordinary regression analysis, probability theory is used to model random errors, and the result is presented as an ordinary regression equation. On the other hand, fuzzy set theory can be used to model fuzzy errors, and the result can be presented using a fuzzy regression equation.

If the phenomenon under study is economic or social, observations are a consequence of the interaction between the beliefs and expectations of the agents who take part in the phenomenon. We have already stated that, in our opinion, such a phenomenon should not be modelled using probability theory. For example, the security prices that are negotiated in financial markets are the consequence of the participants' expectations about the economic future, the trust that the security issuers generate in operators, etc.

In this case the linearity between the explained variable and the explanatory variables, which is assumed using both conventional and fuzzy regression, is oversimplified. However, we believe it is more realistic to model the bias that can arise between the realizations of the dependent variable and their theoretical values on the assumption that the relationship between the dependent variable and the explanatory variables is fuzzy, and not on the assumption that this bias is of a random nature. With respect to the prices of financial assets, we will be assuming, at least, that there is a strong subjective component in their determination.

Moreover, in many circumstances the observations of the dependent variable, the independent variable or both do not come from a particular number, but from a confidence interval. For example, the price that is negotiated in financial markets during a trading day for a particular security is hardly unique, but it is usually negotiated within a range limited by a maximum price and a minimum price. When the minimum square techniques—or the most sophisticated likelihood—are used, the observations of the explained (and explanatory) variable must be quantified using a unique number (for example, the average price negotiated or the last price in the model which will be implemented). This procedure clearly involves considerable information loss. When fuzzy regression models are implemented, the value of the observed variables does not need to be reduced to and represented by a single real number so we can work with all the information available.

The fuzzy regression method uses linear programming to estimate the fuzzy coefficients in the resulting models. As pointed out in [44], as the number of data sets increases, so it may be more difficult to use linear programming to estimate fuzzy beta. Each data set results in two constraints on the fuzzy regression formulation. As the number of data sets increases, the number of constraints increases proportionally. This increase might result in computational difficulties when using linear programme software or computers.

Taking into account that econometric fuzzy models mean that all the stock quotes information can be incorporated, and that there is no need to make assumptions on the basis of the random term which is difficult to apply, this method makes it possible to improve the prediction of future stock quotes.

The empirical evidence obtained from fuzzy regressions is consistent with that reported in traditional econometric studies on beta stability. The relevance of this verification is that the more stable β is, the more confident the predictions are. We observe that sector betas are more stable than subsector betas. Additionally, betas are more stable if the estimation periods are longer. Moreover, some sectors are more stable than others, independently of the estimation period. Consumer goods and Consumer Services are the most stable sectors, whereas Technology and Communications is highly unstable.

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Appendix A. Quarterly beta

Table 3. Oil and Energy

Quarter	Oil		Electricity and Gas		Water and Others		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	0.69	0.69	1.53	1.53	-0.15	0.87	1.23	1.23
2 2005	1.48	1.48	0.95	0.95	1.17	1.17	1.27	1.27
3 2005	-0.62	-0.62	1.20	1.29	1.90	1.90	0.65	0.65
4 2005	2.30	2.30	1.32	1.32	0.87	0.87	1.46	1.46
1 2006	-2.32	0.02	0.74	2.20	3.68	3.68	-0.28	1.62
2 2006	1.78	1.78	0.80	0.80	0.35	0.35	1.02	1.02
3 2006	0.51	1.12	1.74	1.74	0.75	0.75	1.55	1.55
4 2006	1.84	2.12	0.46	0.46	-0.22	-0.22	0.62	0.62
1 2007	1.13	1.13	0.32	0.32	1.21	1.21	0.25	0.25
2 2007	1.13	1.13	1.04	1.04	1.04	1.29	1.22	1.22
3 2007	2.02	2.02	0.81	0.81	0.44	0.44	1.39	1.39
4 2007	1.00	1.00	0.24	0.24	0.33	0.80	0.31	0.31
1 2008	-0.52	-0.52	0.41	0.41	-0.01	-0.01	1.34	1.34
2 2008	0.29	0.29	0.03	0.03	-0.10	-0.10	0.57	0.57
3 2008	0.36	0.36	-0.35	-0.35	0.03	0.03	0.22	0.22
4 2008	-0.41	-0.41	-0.15	-0.15	0.16	0.16	0.80	0.80
1 2009	0.14	0.14	0.02	0.02	0.69	0.69	0.58	0.58
2 2009	-0.41	-0.41	-0.09	-0.09	-0.12	-0.12	0.62	0.62

Table 4. Basic Material, Industry and Construction

Quarter	Mineral/ Metals/ Transf		Capital Goods		Construction		Construct. Materials		Chemical		Engineering and Others		Aerospace		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	1.17	2.09	0.49	0.49	0.93	0.93	0.92	0.92	1.59	1.59	0.66	0.66	0.95	0.95	0.70	0.81
2 2005	1.41	1.41	0.02	0.02	1.39	1.39	-0.37	-0.37	4.06	4.06	0.59	0.59	1.16	1.16	1.13	1.13
3 2005	0.79	0.79	0.86	1.36	0.49	0.93	-0.07	0.33	-2.41	4.65	1.45	1.45	0.27	0.27	0.79	0.84
4 2005	0.66	0.66	1.57	1.57	1.88	1.88	1.06	1.06	1.64	1.64	0.33	0.33	2.08	2.08	1.76	1.76
1 2006	1.49	6.65	1.61	1.88	1.19	1.19	3.54	3.83	0.39	0.39	2.29	3.12	-0.79	-0.79	1.69	1.69
2 2006	0.84	0.84	1.13	1.13	1.88	1.88	1.40	1.40	0.25	0.25	2.95	2.95	1.57	1.57	1.24	1.24
3 2006	0.26	0.26	0.66	0.66	1.03	1.27	0.05	0.05	-0.17	-0.17	1.54	1.54	-0.06	-0.06	1.01	1.12
4 2006	0.64	0.64	0.47	0.47	1.23	1.23	0.65	0.65	0.50	2.39	0.95	0.95	-2.31	-0.25	0.88	0.88
1 2007	0.05	0.05	1.02	1.02	0.91	0.91	1.34	1.34	1.00	1.00	0.44	0.44	0.04	0.04	0.87	0.87
2 2007	1.00	1.00	0.55	0.55	1.73	1.73	1.08	1.08	0.60	0.60	1.30	1.30	1.19	1.19	1.12	1.12
3 2007	0.84	0.84	1.28	1.28	2.04	2.04	1.49	1.49	1.45	1.45	1.16	1.16	1.41	1.41	1.45	1.45
4 2007	0.11	0.11	1.78	1.78	0.71	0.71	2.36	2.36	3.52	3.52	0.58	0.58	-0.30	-0.30	0.87	0.87
1 2008	0.74	0.74	0.18	0.18	1.39	1.39	1.38	1.38	0.37	0.37	0.51	0.51	2.13	2.13	1.19	1.19
2 2008	0.10	0.10	0.25	0.25	0.16	0.16	0.42	0.42	0.62	0.62	0.36	0.36	1.72	1.72	0.43	0.43
3 2008	0.71	0.71	0.28	0.28	0.92	0.92	0.64	0.64	0.61	0.61	0.40	0.40	1.78	1.86	0.62	0.62
4 2008	0.07	0.07	0.14	0.14	-0.04	-0.04	-0.19	-0.19	0.09	0.09	0.17	0.17	-0.20	-0.20	0.94	0.94
1 2009	1.00	1.00	1.00	1.00	-0.47	-0.47	0.40	0.40	-1.13	-1.13	0.61	0.61	-1.11	-1.11	0.27	0.27
2 2009	0.59	0.59	0.78	0.78	0.03	0.03	0.59	0.59	1.11	1.11	1.11	1.11	0.93	0.93	0.85	0.85

Table 5. Consumer goods

Quarter	Food and Beverages		Textiles/Clothing/ Shoes		Paper and Graphic Arts		Other Consumer Goods		Pharmacy Products		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	0.63	0.63	1.37	1.37	0.49	0.49	0.67	0.67	1.36	1.36	1.01	1.01
2 2005	0.93	0.93	-0.09	-0.09	0.46	0.46	0.70	0.70	1.74	1.74	0.37	0.37
3 2005	-0.32	-0.32	0.23	0.61	-0.18	-0.18	-0.79	-0.79	-1.26	0.34	0.16	0.78
4 2005	1.15	1.15	0.05	0.05	0.29	0.29	1.00	1.09	-0.36	-0.36	0.76	0.76
1 2006	1.23	1.69	1.23	1.23	-1.07	0.56	-2.09	3.52	0.94	0.94	0.42	0.42
2 2006	0.76	0.76	0.32	0.32	0.79	0.79	0.06	0.81	0.50	0.50	0.55	0.55
3 2006	0.41	0.41	0.68	0.68	1.01	1.01	0.28	0.28	0.82	0.82	0.60	0.60
4 2006	0.59	0.59	1.23	2.39	0.17	0.17	0.31	1.49	-0.46	-0.46	1.12	1.40
1 2007	0.69	0.69	0.62	1.33	2.07	2.07	0.07	0.07	1.48	1.96	0.51	0.51
2 2007	0.82	0.82	1.24	1.24	0.64	0.64	-0.06	-0.06	0.75	0.75	0.59	0.59
3 2007	0.49	0.49	1.11	1.11	1.15	1.15	-0.18	-0.18	0.95	0.95	0.55	0.55
4 2007	0.85	0.85	0.97	0.97	1.10	1.10	-0.04	-0.04	2.63	2.63	0.50	0.50
1 2008	0.91	0.91	0.04	0.04	0.68	0.68	0.00	0.00	0.58	0.58	0.31	0.31
2 2008	0.16	0.16	0.27	0.27	0.48	0.48	0.04	0.04	-0.15	-0.15	0.11	0.11
3 2008	0.53	0.53	-0.13	-0.13	0.67	0.67	-0.22	-0.22	0.34	0.34	0.46	0.46
4 2008	0.33	0.33	0.95	0.95	0.56	0.56	0.32	0.32	0.55	0.55	0.60	0.60
1 2009	0.20	0.20	1.25	1.25	0.53	0.53	0.16	0.16	0.78	0.78	0.91	0.91
2 2009	-0.18	-0.18	-0.04	1.31	0.71	0.71	0.30	0.30	0.98	0.98	0.31	0.86

Table 6. Consumer services

Quarter	Leisure/Tour. Hotel		Retailing		Communication and Publicity		Car Parks/ Motorways		Transport/ Distribution		Other Services		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	0.18	0.48	3.54	3.54	-0.42	-0.42	0.11	0.69	-1.37	1.16	-1.22	-0.53	0.4	0.45
2 2005	0.29	0.29	0.28	0.28	1.83	1.83	1.43	1.43	1.09	1.09	0.99	0.99	0.9	0.91
3 2005	0.13	0.13	-0.31	0.77	0.58	0.58	-0.25	-0.25	1.49	1.49	0.82	0.91	0.3	0.32
4 2005	0.68	0.68	0.23	0.23	2.31	2.31	1.70	1.70	0.52	0.52	2.25	2.25	1.5	1.54
1 2006	1.33	2.48	2.13	2.13	0.92	0.92	0.50	0.50	0.92	0.92	-1.14	-0.85	0.6	0.61
2 2006	0.43	0.43	1.78	1.78	0.78	0.78	0.68	0.68	0.47	0.47	0.46	0.46	0.8	0.81
3 2006	0.56	0.56	1.98	1.98	0.65	0.92	0.36	0.36	0.64	0.64	1.64	1.64	0.6	0.81
4 2006	0.48	0.48	-0.31	-0.31	0.47	0.47	0.47	1.03	0.77	0.77	-0.23	-0.23	0.5	0.57
1 2007	1.21	1.21	0.99	0.99	0.31	0.31	0.60	0.60	1.41	2.17	0.95	0.95	0.8	0.81
2 2007	0.54	0.54	2.28	2.28	0.99	0.99	0.90	0.90	0.98	0.98	0.87	0.87	0.7	0.73
3 2007	1.40	1.40	2.23	2.23	0.60	0.60	1.73	1.73	0.46	0.46	1.05	1.05	1.2	1.26
4 2007	1.97	3.06	1.45	1.45	0.58	0.58	1.03	1.03	1.85	1.85	1.15	1.15	0.9	0.94
1 2008	0.51	0.51	-0.54	-0.54	0.45	0.45	0.86	0.86	0.76	0.76	1.33	1.33	1.0	1.01
2 2008	0.35	0.35	-0.14	-0.14	0.48	0.48	0.79	0.79	0.63	0.63	0.03	0.03	0.6	0.65
3 2008	1.11	1.11	0.65	0.65	0.50	0.50	0.84	0.84	2.81	2.81	-0.06	-0.06	0.8	0.85
4 2008	0.84	0.84	0.91	0.91	0.49	0.49	0.70	0.70	0.83	0.83	0.48	0.75	0.6	0.65
1 2009	0.98	0.98	1.25	1.25	1.02	1.02	0.78	0.78	0.75	0.75	-0.43	-0.43	0.5	0.56
2 2009	2.39	2.39	0.88	1.93	1.59	2.21	0.48	0.74	0.88	0.88	0.93	0.93	0.9	1.37

Table 7. Financial service & real estate

Quarter	Banks		Insurance		Portfolio And Holding		Real Estate And Others		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	0.96	0.96	0.98	1.48	-0.11	-0.11	0.89	0.89	0.97	0.97
2 2005	1.17	1.17	1.37	1.37	1.07	1.07	0.43	0.43	1.23	1.23
3 2005	0.75	1.50	1.08	1.08	0.51	1.30	-0.53	-0.53	0.79	1.42
4 2005	0.81	0.81	0.78	0.78	1.79	1.79	0.92	0.92	0.88	0.88
1 2006	1.18	1.53	0.67	1.07	1.01	1.93	-1.77	-1.77	1.38	1.38
2 2006	0.78	0.78	0.78	0.78	1.16	1.26	0.95	0.95	0.90	0.90
3 2006	0.79	0.79	0.62	0.62	1.20	1.20	0.24	0.24	0.81	0.81
4 2006	0.96	1.66	1.05	1.05	0.57	0.57	1.45	1.45	0.82	1.48
1 2007	1.05	1.05	0.63	0.63	1.22	1.22	1.46	1.46	1.01	1.01
2 2007	0.88	0.88	0.66	0.66	0.45	0.45	1.39	1.39	0.89	0.89
3 2007	0.87	0.87	0.75	0.75	1.46	1.46	0.18	0.18	0.82	0.82
4 2007	0.29	1.10	1.52	2.06	1.09	1.09	0.73	0.73	0.33	1.12
1 2008	0.73	0.88	0.54	0.54	1.59	1.59	0.59	0.59	0.83	0.91
2 2008	1.10	1.10	-0.11	-0.11	0.79	0.79	0.10	0.10	1.03	1.03
3 2008	0.31	0.31	0.50	0.50	0.05	0.05	-0.22	-0.22	0.25	0.25
4 2008	0.98	0.98	0.91	0.91	0.59	0.59	0.56	0.56	0.96	0.96
1 2009	1.84	1.84	1.22	1.22	0.91	0.91	0.38	0.38	1.75	1.75
2 2009	1.04	2.18	0.89	1.93	0.43	0.52	0.76	0.76	1.00	2.09

Table 8. Technology & Telecommunications

Quarter	Electronics/Software		Telecommunications & Others		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	0.30	0.30	0.89	0.89	0.90	0.90
2 2005	1.15	1.15	1.34	1.34	1.36	1.36
3 2005	1.59	1.59	0.55	0.55	0.60	0.60
4 2005	1.36	1.36	-0.89	-0.89	-0.76	-0.76
1 2006	0.75	1.49	0.97	1.68	1.01	1.74
2 2006	0.91	0.91	0.84	0.84	0.82	0.82
3 2006	0.86	1.23	0.93	0.93	0.95	0.95
4 2006	0.60	0.60	0.96	0.96	0.98	0.98
1 2007	1.08	1.09	0.95	0.95	0.94	0.94
2 2007	0.87	0.87	0.67	0.74	0.68	0.75
3 2007	1.17	1.17	0.95	0.95	0.93	0.93
4 2007	2.20	2.20	0.31	3.54	0.35	3.46
1 2008	0.49	0.49	1.23	1.23	1.22	1.22
2 2008	0.55	0.55	0.33	0.33	0.33	0.33
3 2008	0.37	0.37	0.13	0.13	0.12	0.12
4 2008	0.54	0.54	0.81	0.81	0.82	0.82
1 2009	0.18	0.18	0.15	0.15	0.15	0.15
2 2009	0.41	0.41	0.34	0.40	0.33	0.40

Appendix B. Biannual beta

Table 9. Oil and energy

Semester	Oil		Electricity And Gas		Water And Others		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	1.30	1.30	1.53	1.53	0.24	0.24	1.27	1.27
2 2005	1.12	1.12	1.29	1.29	1.90	1.90	1.27	1.27
1 2006	-0.64	-0.64	2.03	2.03	1.83	1.83	1.60	1.60
2 2006	0.79	0.79	1.56	1.56	1.70	1.70	1.43	1.43
1 2007	1.13	1.13	0.88	0.88	1.29	1.29	1.04	1.04
2 2007	1.08	1.08	0.81	0.81	0.44	0.44	1.17	1.17
1 2008	-0.57	-0.57	0.41	0.41	-0.10	-0.10	1.27	1.27
2 2008	0.00	0.00	-0.15	-0.15	0.16	0.16	0.80	0.80
1 2009	0.11	0.11	0.02	0.02	0.02	0.02	0.58	0.58

Table 10. Basic material, industry and construction

Semester	Mineral/ Metals/Transf		Capital Goods		Construction		Construct. Materials		Chemical		Engineering and Others		Aerospace		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	1.41	2.09	0.15	0.15	-0.37	-0.37	-0.37	-0.37	1.59	1.59	0.59	0.59	0.95	0.95	0.70	0.70
2 2005	0.66	0.66	1.33	1.33	0.69	0.69	0.69	0.69	4.65	4.65	0.42	0.42	0.70	0.70	1.47	1.47
1 2006	2.40	2.40	1.13	1.13	2.33	2.33	2.33	2.33	-0.13	-0.13	2.95	2.95	1.57	1.57	1.24	1.24
2 2006	0.21	0.21	0.15	0.15	0.05	0.05	0.05	0.05	0.13	0.13	1.13	1.13	-0.67	-0.67	0.88	0.88
1 2007	0.91	0.91	0.70	0.70	-0.30	-0.30	-0.30	-0.30	1.41	1.41	0.44	0.44	1.19	1.19	1.12	1.12
2 2007	0.84	0.84	0.70	0.70	1.49	1.49	1.49	1.49	2.40	2.40	1.16	1.16	-0.30	-0.30	1.45	1.45
1 2008	0.47	0.47	-0.18	-0.18	1.38	1.38	1.38	1.38	0.37	0.37	0.51	0.51	2.26	2.26	1.06	1.06
2 2008	0.25	0.25	0.19	0.19	0.31	0.31	0.31	0.31	0.40	0.40	0.29	0.29	-0.03	-0.03	0.94	0.94
1 2009	1.00	1.00	1.00	1.00	0.40	0.40	0.40	0.40	-0.28	-0.28	1.10	1.10	-1.11	-1.11	0.27	0.27

Table 11. Consumer goods

Semester	Food and Beverage		Textiles/ Clothing/Shoes		Paper and Graphic Arts		Other Consumer Goods		Pharmacy Products		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	0.80	0.80	0.09	0.09	0.69	0.69	0.70	0.70	1.36	1.36	0.66	0.66
2 2005	-0.09	-0.09	0.23	0.23	0.01	0.01	0.41	0.41	0.34	0.34	0.40	0.40
1 2006	0.58	0.58	0.32	0.32	0.74	0.74	-1.00	-1.00	0.50	0.50	0.55	0.55
2 2006	0.59	0.59	1.23	1.23	0.90	0.90	0.61	0.61	-0.44	-0.44	0.99	0.99
1 2007	0.74	0.74	1.18	1.18	1.68	1.68	0.07	0.07	1.96	1.96	0.51	0.51
2 2007	0.49	0.49	1.11	1.11	1.15	1.15	-0.18	-0.18	0.95	0.95	0.55	0.55
1 2008	0.87	0.87	0.04	0.04	0.48	0.48	0.00	0.00	0.58	0.58	0.31	0.31
2 2008	0.52	0.52	0.95	0.95	0.56	0.56	0.32	0.32	0.55	0.55	0.60	0.60
1 2009	0.20	0.20	1.13	1.13	0.71	0.71	0.22	0.22	0.78	0.78	0.91	0.91

Table 12. Consumer services

Semester	Leisure/ Tour. Hotel		Retail		Communic. Publicity		Car Parks /Motorways		Transport/ Distribution		Other Services		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	0.32	0.32	2.76	2.76	0.72	0.72	1.43	1.43	1.16	1.16	-0.53	-0.53	1.20	1.20
2 2005	0.43	0.43	0.66	0.66	0.88	0.88	1.90	1.90	0.42	0.42	2.25	2.25	1.07	1.07
1 2006	0.43	0.43	2.13	2.13	0.55	0.55	0.72	0.72	0.47	0.47	0.08	0.08	0.78	0.78
2 2006	0.79	0.79	0.98	0.98	0.65	0.65	0.47	0.47	0.64	0.64	1.49	1.49	0.60	0.60
1 2007	0.54	0.54	2.28	2.28	0.48	0.48	0.90	0.90	2.17	2.17	1.00	1.00	0.81	0.81
2 2007	1.40	1.40	2.23	2.23	0.60	0.60	1.55	1.55	1.03	1.03	1.15	1.15	1.26	1.26
1 2008	0.51	0.51	-0.54	-0.54	0.45	0.45	1.20	1.20	0.43	0.43	0.90	0.90	1.01	1.01
2 2008	0.84	0.84	0.91	0.91	0.44	0.44	0.84	0.84	0.75	0.75	0.49	0.75	0.61	0.61
1 2009	0.98	0.98	1.25	1.25	1.14	1.14	0.78	0.78	0.75	0.75	-0.22	-0.22	1.00	1.00

Table 13. Financial service & real estate

Semester	Banks		Insurance		Portfolio And Holding		Real Estate And Others		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	0.96	0.96	1.10	1.10	-0.11	-0.11	0.89	0.89	0.97	0.97
2 2005	0.85	0.85	1.61	1.61	0.37	0.37	1.45	1.45	0.82	0.82
1 2006	0.87	0.87	1.07	1.07	1.16	1.16	0.95	0.95	0.90	0.90
2 2006	0.96	0.96	0.92	0.92	1.11	1.11	1.31	1.31	0.82	0.82
1 2007	1.05	1.05	2.03	2.03	0.92	0.92	1.39	1.39	1.04	1.04
2 2007	0.94	0.94	1.09	1.09	1.46	1.46	0.18	0.18	0.91	0.91
1 2008	0.88	0.88	0.42	0.42	1.43	1.43	0.41	0.41	0.91	0.91
2 2008	0.98	0.98	0.91	0.91	0.59	0.59	0.56	0.56	0.95	0.95
1 2009	1.84	1.84	1.70	1.70	0.72	0.72	0.38	0.38	1.75	1.75

Table 14. Technology & telecommunications

Semester	Electronics/Software		Telecommunications & Others		SECTOR	
	β_1	β_2	β_1	β_2	β_1	β_2
1 2005	0.34	0.34	1.19	1.19	1.18	1.18
2 2005	1.36	1.36	-0.10	-0.10	-0.02	-0.02
1 2006	0.75	0.75	0.97	0.97	1.01	1.01
2 2006	0.60	0.60	0.93	0.93	0.95	0.95
1 2007	1.02	1.02	0.83	0.83	0.81	0.81
2 2007	1.17	1.17	2.12	2.12	2.04	2.04
1 2008	0.49	0.49	1.12	1.12	1.09	1.09
2 2008	0.54	0.54	0.84	0.84	0.80	0.80
1 2009	0.41	0.41	0.15	0.15	0.15	0.15

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